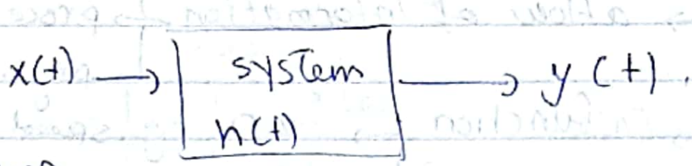
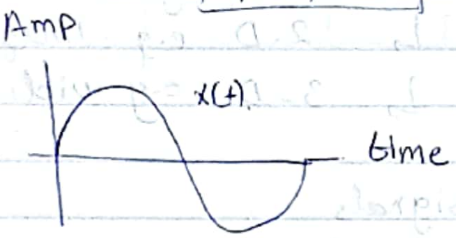
Dsp: digital signal processing:

Continues system:

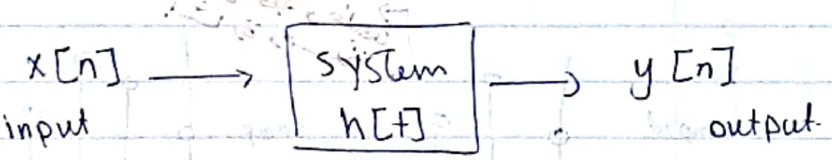


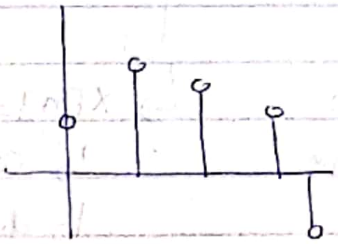


⇒ Mathamatical Models DFE

$$\tau \frac{dy(t)}{dt} + y(t) = x(t).$$

Discrete system





, n: integer number.

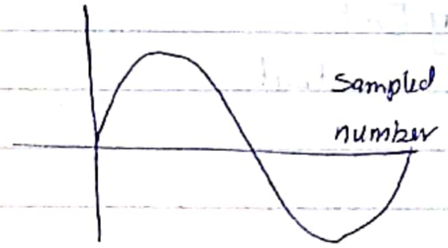
CH2: Discrete-time signal and system

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

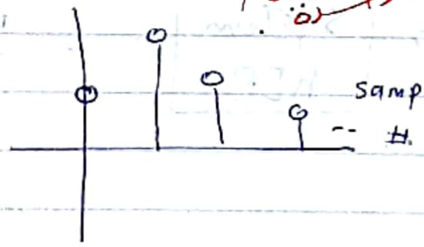
signals:
- a flow of Information
  - event
  - process
  - physical
- function.
  - 1-D. e.g speed.
  - 2-D e.g Image
  - 3-D e.g vidio.

signals.
- continues-time signals
- discrete-time signals
- digital signals.

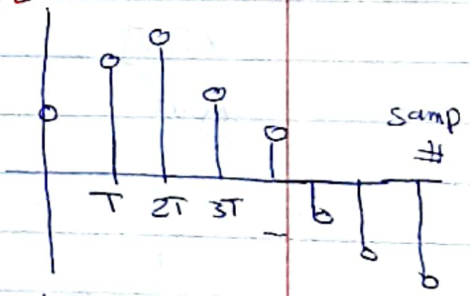
يوحد الاشارات
المتشابهة وعليها
اشارة وحدة.



- cont. Amp.
- cont. time.



- Dis. Amp
- Dis. time



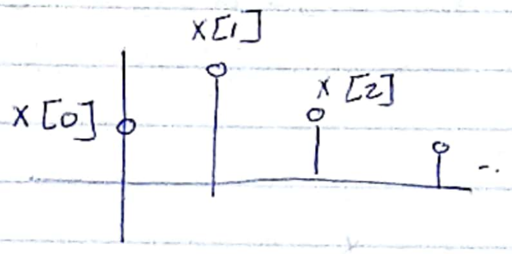
- $x[n] = x(nT_s)$
  - cont. Amp.
  - disc. time.

Discrete Signal

$x[n] = x(nT)$

n: integer value

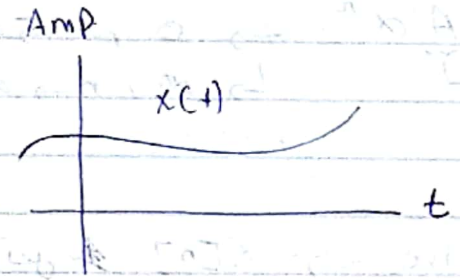
T: sampling period.

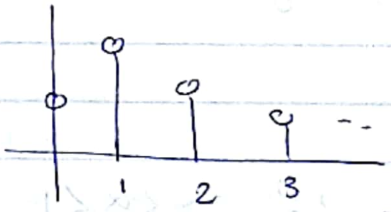


$$x[n] = \sum_{K=-\infty}^{\infty} x[K]\,\delta[n-K]$$

Basic sequences:

continues | **Discrete**





n: int

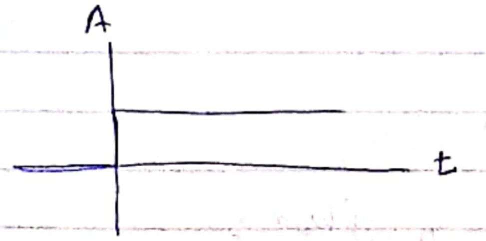
Ts: samplin

periodic

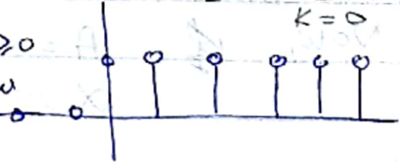
1) unit step function.

1) unit step sequences.

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

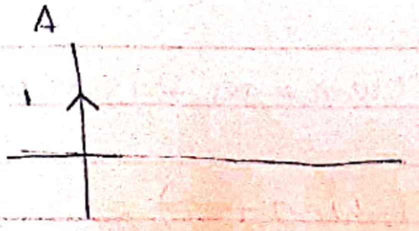


$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & o.w \end{cases}$$



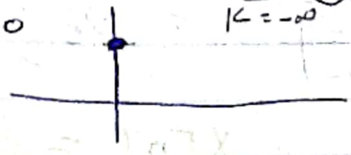
2) unit Impulse function

2) unit sample sequence

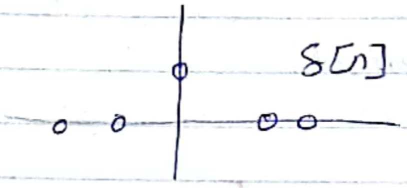


$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & o.w \end{cases}$$

$$\delta[n] = \sum_{K=-\infty}^{\infty} x[K]\,\delta[n-K]$$



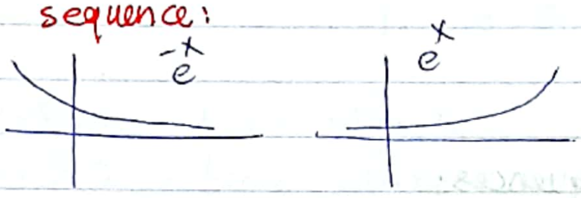
Example: ⟹ Express the unit sample sequence in terms of unit step sequence



$\Rightarrow \delta[n] = u[n] - u[n-1]$

3) Exponential sequence:

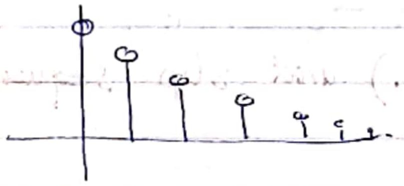
continues:



⟹ In general: ⟹ $x[n] = \underset{\text{continues}}{A\,\alpha^{n}}$ $\rightarrow 0,\ n \rightarrow \infty$

$\hookrightarrow \infty,\ n \rightarrow \infty$

- If $0 < \alpha < 1$, A: positive ⟹ $x[n]$ تقل



Now,

$$A = |A|\,e^{j\phi}$$
$$\alpha = |\alpha|\,e^{j\omega_0}$$

$$x[n] = Re\{|A|\,e^{j\phi} \cdot |\alpha|^{n}\,e^{j\omega_0 n}\}$$

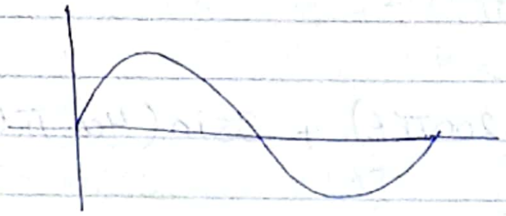
$$= Re\{|A|\,|\alpha|\,e^{j(\omega_0 n + \phi)}\}$$

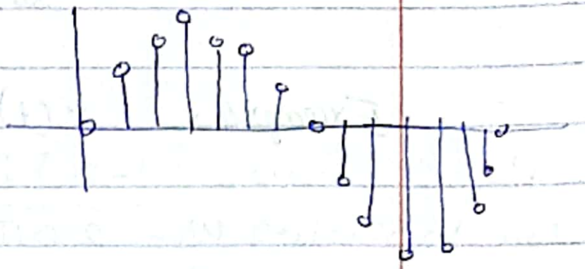
$\boxed{\text{If } |\alpha| = 1}$ $= Re\{|A|\,(\cos(\omega_0 n + \phi) + j\sin(\omega_0 n + \phi)\}$

$$x[n] = |A|\,\cos(\omega_0 n + \phi)$$

continues: **Discreta:**

$x(t) = |A| \cos(w_0 t + \phi)$ $x[n] = |A| \cos(w_0 n + \phi)$





periodic and aperiodic:

To check if the signal is periodic oR aperiodic

$\Rightarrow x[t+T] \overset{?}{=} x(t)$ → yes : periodic
→ No : aperiodic

continues:

Example: $x(t) = 3\cos(200\pi t)$

$w_0 = 200\pi$ $x(t+T) = 3\cos(200\pi t + 200\pi T)$

$2\pi f_0 = 200\pi$ $= 3\cos(200\pi t + 200\pi \cdot \frac{1}{100})$

$f_0 = 100$ $= 3\cos(200\pi t + 2\pi)$

$\boxed{T = 1/100}$

↳ $\cos(\alpha \mp \beta) = \cos(\alpha)\cos(\beta) \pm \sin(\alpha)\sin(\beta)$

$= 3\cos(200\pi t)\cos(2\pi) - \sin(200\pi t)\sin(2\pi)$

$= 3\cos(200\pi t)$

$= x(t)$

↳ periodic //

Example: $x(t) = 3\sin(15t)$

$$\omega_0 = 15$$
$$2\pi f_0 = 15$$
$$\rightarrow f_0 = 15/2\pi \text{ Hz}$$
$$T = 2\pi/15 \text{ s}$$

Excample: $x(t) = 2\cos(200\pi t) + 3\sin(400\pi t)$

$\omega_1 = 200\pi$ $\qquad$ $\omega_2 = 400\pi$

$2\pi f_1 = 200\pi$ $\qquad$ $2\pi f_2 = 400\pi$

$\boxed{f_1 = 100 \text{ Hz}}$ $\qquad$ $\boxed{f_2 = 200 \text{ Hz}}$

$$\frac{n_1 f_0}{n_2 f_0} = \frac{100}{200} = \frac{1}{2} = \frac{n_1}{n_2} \quad \frac{\text{int}}{\text{int}}$$ [periodic] ✓

when $n_1 = 1 \Rightarrow \boxed{f_0 = 100 \text{ Hz}}$ fundamental frequency

Example: $x(t) = \sin\left(\frac{10\pi t}{3}\right) + \sin\left(\frac{5\pi t}{4}\right)$.

$\omega_1 = 10\pi/3$. $\qquad$ $\omega_2 = 5\pi/4$

$2\pi f_1 = 10\pi/3$ $\qquad$ $2\pi f_2 = 5\pi/4$

$f_1 = \frac{5}{3}$ Hz. $\qquad$ $f_2 = 5/8$ Hz

القاسم المشترك الأكبر $\quad GCD(f_1, f_2) = GCD\left(\frac{5}{3}, \frac{5}{8}\right) = \frac{5}{24}$

⇒ To check the periodic signal in case discrete signal:

$$x[n] = |A| \cos(\omega_0 n + \Phi)$$

fundamental period $\quad x[n+N] = |A| \cos[\omega_0(n+N) + \Phi]$

$$= |A| \cos(\omega_0 n + \omega_0 N + \Phi)$$
$$= |A| \cos(\underbrace{\omega_0 n + \Phi}_{\alpha} + \underbrace{\omega_0 N}_{\beta})$$

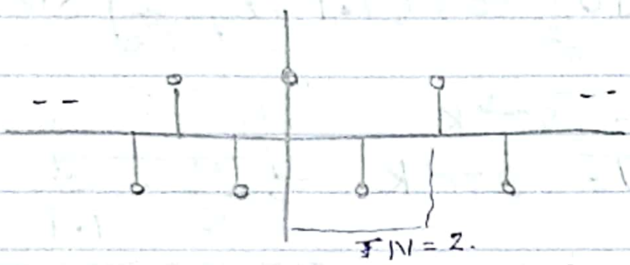
$$= |A| \left[\cos(w_0 n + \Phi) \underbrace{\sin(w_0 N)}_{1} - \sin(w_0 n + \Phi) \underbrace{\sin(w_0 N)}_{0} \right]$$

↳ To acheive the periodicty of the Signal

$\Rightarrow$ $w_0 N = 2\pi K$; N, K int Number.

$\Rightarrow$ $\frac{N}{K} = \frac{2\pi}{w_0}$; rational #

Example: $x[n] = \sum_{K=-\infty}^{\infty} (-1)^n \delta[n-k]$. check if $x[n]$ is periodic oR not.



periodic
$N = 2.$

Example: $x[n] = 3\cos(0.2\pi n)$

$0.2\pi N = 2\pi K \Rightarrow \frac{N}{K} = \frac{2}{0.2} = \frac{10}{1}$

↳ The signal is periodic if $\boxed{N = 10, \quad K = 1}$

Example: $x[n] = 5\sin(0.3\pi n)$

$0.3\pi N = 2\pi K \Rightarrow \frac{N}{K} = \frac{2}{0.3} = \frac{20}{3}$

↳ The signal is periodic if $\boxed{N = 20, \quad K = 3}$

Example: $x[n] = 4\cos(0.5n)$

$0.5 N = 2\pi K \Rightarrow \frac{N}{K} = \frac{2\pi}{0.5} \Rightarrow$ aperiodic signal.

example: $x[n] = 8\sin\left(\frac{\pi n}{\sqrt{2}}\right)$

$$\omega_0 N = 2\pi K$$

$$\frac{\pi}{\sqrt{2}} N = 2\pi K \Rightarrow \frac{N}{K} = \frac{2\sqrt{2}}{1}$$

↳ aperiodic signal.

Example: $x[n] = 10 e^{-j 1.1 \pi n}$

$$x[n] = 10\left[\cos(1.1\pi n) + j\sin(1.1\pi n)\right]$$

$$\omega_0 N = 2\pi K$$

$$1.1\pi N = 2\pi K \Rightarrow \frac{N}{K} = \frac{2}{1.1} = \frac{20}{11}$$

↳ periodic, if $\boxed{N = 20,\ K = 11}$

Example: $x[n] = 10\, e^{-j 1.1 \pi (n-1)}\, \underbrace{u(n-1)}$

↳ aperiodic;

$$x[n] = \begin{cases} 10 e^{-j 1.1 \pi (n-1)} & , n \geq 1 \\ 0 & , o.w \end{cases}$$

رح يكون
في فترة صفر
وفترة مضروبة
بـ 1، بالتالي
الاشارة لا تكرر نفسها

↳ In Continues Signals:

$$x(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \sin(\omega_3 t)$$

قاسم مشترك اكبر ↳ $f_0 = GCD(f_1, f_2, f_3)$

مضاعف مشترك أصغر $T_0 = LCM(T_1, T_2, T_3)$

↳ In Discrete Signals:

$$x[n] = x_1[n] + x_2[n] + x_3[n]$$

↳ $N = LCM(N_1, N_2, N_3)$

Example: check the periodicity of each signal in case of periodic signal, specify the value of K and N.

1) $x[n] = 4\cos(0.2\pi n) - 3\sin(0.3\pi n) + 5\cos(0.4\pi n)$

$w_1 N_1 = 2\pi k_1$

$0.2\pi N_1 = 2\pi k_1$

$\frac{N_1}{K_1} = \frac{2}{0.2} = \frac{10}{1}$

$\boxed{N_1 = 10,\ K_1 = 1}$

$w_2 N_2 = 2\pi k_2$

$0.3\pi N_2 = 2\pi k_2$

$\frac{N_2}{K_2} = \frac{2}{0.3} = \frac{20}{3}$

$\boxed{N_2 = 20,\ K_2 = 3}$

$w_3 N_3 = 2\pi k_3$

$0.4\pi N_3 = 2\pi k_3$

$\frac{N_3}{K_3} = \frac{2}{0.4}$

$\boxed{N_3 = 5,\ K_3 = 1}$

$N = LCM(N_1, N_2, N_3)$

$= LCM(10, 20, 5) = 20$

10	$\boxed{20}$	5
$\boxed{20}$	40	10
30	60	15
⋮	⋮	$\boxed{20}$

2) $x[n] = 3\sin(\frac{\pi}{4}n) + 5\cos(\frac{\pi}{3}n) - 7\sin(\frac{\pi}{2}n)$

$N_1 = 8$, $K_1 = 1$

$N_2 = 6$, $K_2 = 1$

$N_3 = 4$, $K_3 = 1$

$N = LCM(8, 6, 4) = 24$

$f_0 = GCD(8, 6, 4) = 2$

3) $x[n] = 2\cos(\sqrt{2}\pi n) + 5\sin(2\sqrt{2}\pi n)$

$w_1 N_1 = 2\pi k_1$

$\Rightarrow \frac{N_1}{K_1} = \frac{2}{\sqrt{2}} \rightarrow$ aperiodic

4) $x[n] = 10\cos(0.1\pi n) - 6\cos(0.9\pi n) + 5\sin(0.7n)$

$$\omega_3 N_3 = 2\pi k_3$$
$$0.7\, N_3 = 2\pi k_3$$
$$\frac{N_3}{k_3} = \frac{2\pi}{0.7} \rightarrow \text{aperiodic}$$

Discrete time system :=))

$$x[n] \longrightarrow \boxed{T\{x[n]\}} \longrightarrow y[n]$$

1) Linearity.

↳ Continues :=))

Example: $y(t) = \log_{10}(x(t))$

$$\alpha_1 y_1(t) = \log_{10}(\alpha_1 x_1(t))$$
$$\alpha_2 y_2(t) = \log_{10}(\alpha_2 x_2(t))$$

$$\alpha_1 y_1(t) + \alpha_2 y_2(t) = \log_{10}(\alpha_1 x_1(t)) + \log_{10}(\alpha_2 x_2(t)) \quad \text{—} ①$$

Assume: $\alpha_3 x_3(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$

$\alpha_3 y_3(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$

$$\alpha_3 y_3(t) = \log_{10}(\alpha_3 x_3(t))$$

$$\alpha_1 y_1(t) + \alpha_2 y_2(t) = \log_{10}(\alpha_1 x_1(t) + \alpha_2 x_2(t)) \quad \text{—} ②$$

$$Eq(1) \neq Eq(2)$$

↳ Non-linear system.

Discrete !=))

$$x[n] \longrightarrow \boxed{T\{x[n]\}} \longrightarrow y[n]$$

Example: $y[n] = \log_{10}(x[n])$

to check the linearity → scalling
→ Adding.

$$\alpha_1 y_1[n] = \log_{10}(\alpha_1 x_1[n])$$
$$\alpha_2 y_2[n] = \log_{10}(\alpha_2 x_2[n])$$

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \log_{10}(\alpha_1 x_1[n]) + \log_{10}(\alpha_2 x_2[n]) \text{ — ①}$$

Assume: $\alpha_3 x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$
$\alpha_3 y_3[n] = \alpha_1 y_1[n] + \alpha_2 y_2[n]$

$$\alpha_3 y_3[n] = \log_{10}(\alpha_3 x_3[n])$$

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \log_{10}(\alpha_1 x_1[n] + \alpha_2 x_2[n]) \text{ — ②}$$

$$Eq(1) \neq Eq(2)$$
↳ non-linear system

Examples: consider the following accumulator system which is defined by the following Input, output equation.

$$y[n] = \sum_{K=-\infty}^{\infty} x[k],$$ check the linearity of the system

$$\Rightarrow \alpha_1 y_1[n] = \sum_{K=-\infty}^{n} \alpha_1 x_1[k]$$

$$\alpha_2 y_2[n] = \sum_{K=-\infty}^{n} \alpha_2 x_2[K]$$

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \sum_{K=-\infty}^{n} \alpha_1 x_1[K] + \alpha_2 x_2[K] \quad — ①$$

Assume:

$$\alpha_3 y_3[n] = \alpha_1 y_1[n] + \alpha_2 y_2[n].$$

$$\alpha_3 x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n].$$

$$\alpha_3 y_3[n] = \sum_{K=-\infty}^{n} \alpha_3 x_3[K]$$

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \sum_{K=-\infty}^{n} \alpha_1 x_1[n] + \alpha_2 x_2[n] \quad — ②$$

$Eq(1) = Eq(2) \Rightarrow$ linear system

Time variant and Time invariant.

↳ Continues ε=)).

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$
- → shift time
- → delay of function with respect to Input.

Example: $y(t) = x(M(t))$

- Shift time → $y(t-t_0) = x(M(t-t_0))$ — ①
- Shift function → $y(t-t_0) = x(Mt - t_0)$ — ②

$Eq(1) \neq Eq(2) \Rightarrow$ time variant.

Discrete time system.

- properties of Discrete -Time system.

لم يعني أي نقطة انا بشوف في ـــ [الحاضر] ← بعتمد على حاجة في المستقبل

1) Memory and Memory less هي بس لازم تكون ب Input output

ـــ بالاعتماد على قيمة حالية

e.g. $y[\underline{n}] = (x[\underline{n}])^2$ Memory less.

↑ present present

Example: consider the following accumulator system which is defind by.

$$y[n] = \sum_{k=-\infty}^{n} x[k].$$, ~~check~~

↳ Memory ⇒ لأنه يحتفظ على أكثر (Dinamic).

من حالة بسبب وجود اشارة ال sum

Example: Consider the compresser system which is defined by.

$$T\{x[n]\} = x[Mn]$$

shift time ⇒ $T\{x[n-n_0]\} = x[M(n-n_0)]$ —(1)

= function ⇒ $T\{x[n-n_0]\} = x[Mn-n_0]$ —(2)

$$Eq(1) \neq Eq(2) \Rightarrow \text{varient}$$

Example: Consider the following accumulator system which is defined by

$$T\{x[n]\} = \sum_{K=-\infty}^{n} x[k].$$, check if the system is time variant or not

$\Rightarrow$ time shift: $y[n-n_o] = \sum x[K]$ —①

shift function: $y[n-n_o] = \sum_{K=-\infty}^{n} x[\underbrace{K-n_o}_{u}]$ —②

نعوض

if $u = K - n_o$

$\left[\begin{array}{l} \text{when } K = -\infty \Rightarrow u = -\infty \\ \text{when } K = n \Rightarrow u = n - n_o. \end{array}\right.$

$\Rightarrow \sum_{K=-\infty}^{n-n_o} x[u]$ — ③

Eq (1) = Eq (3) $\Rightarrow$ time-invarient.

المستقبل

↳ **Causal and non-causal**

بعتمد على القيم الماضية والحاضرة

example: consider the following system

$$y[n] = x[n-nd]$$

$$y[n] = x[n-nd] \begin{cases} \to \text{causal} \to nd \geq 0 \\ \to \text{non-causal} \to nd < 0 \end{cases}$$

Example: consider the forward diffrence system defined by the following relationship.

$$y[n] = x[n+1] - x[n]$$

check if the system is causal or non-causal

$\rightarrow \quad y[n] = x[n+1] - x[n] \Rightarrow$ non-causal

قيمة مستقبلية ↑ قيمة حالية

Example: consider the backward diffrence system defined as:

$$y[n] = x[n] - \underbrace{x[n-1]}_{\text{past value}}$$

↳ causal

↳ stability

BIBO

↳ bounded Input. $|x[n]| < M < \infty$

↳ bounded output

Example: check the stability for the following system

$$y[n] = (\overset{x}{\cancel{\alpha}}[n])^2$$

بما انو في تربيع
اذا القيمة رح
تكون محصورة
سيبقى رقمه

$$|y[n]| = |x[n]^2| = (|x[n]|)^2 \leq M < \infty$$

output depend to Input

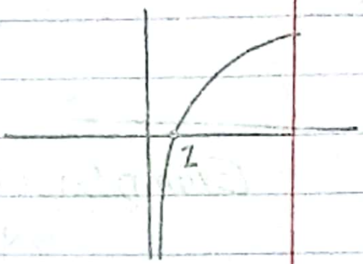
↳ Input ⇒ bounded.

then output bounded

Example: $y[n] = \log(x[n])$

log(0) = ∞ غير معرف

$$|y[n]| = \log(|x[n]|)$$



Since $|x[n]| \leq M < \infty$

↳ هاي ممكن انو تكون صفر.

↳ unstable $|x[n]| = \text{zero}$.

Example: $y[n] = \sum_{K=-\infty}^{n} u[K]$; $u[K]$ unit step.

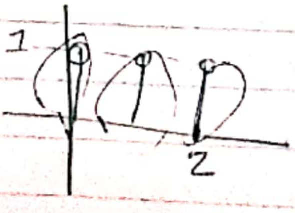
n+1 = عدد القيم

$$= \begin{cases} 0, & n < 0 \\ \circledast\ \boxed{(n+1)}, & n \geq 0. \end{cases}$$

$$|y[n]| = |n+1| \begin{cases} \rightarrow \text{finite} \rightarrow \text{stable} \\ \rightarrow \text{infinite} \rightarrow \text{unstable.} \end{cases}$$

$u[n] =$

$\underline{u[2]},$



$$\Rightarrow u[0] + u[1] + u[2]$$
$$= 1+1+1 = \underline{\underline{3}}$$

Example: For each of the systems, determine whether the system is:

1) stable 2) causal 3) linear
4) time invariant 5) Memoryless

1- $T\{x[n]\} = g[n]\,x[n]$, with $g[n]$ is given.

- **stability [BIBO]**

$$|T\{x[n]\}| = |g[n]\,x[n]|$$

$$\leq |g[n]|\,\underbrace{|x[n]|}_{\uparrow} \; ; \; \underbrace{|x[n]| \leq M < \infty}_{\text{bounded}} \quad \text{BI}$$

↳ The system is stable if $|g[n]| < \infty$

- **causality**

↳ causal, ⇒ depend on the present value of n.

- **linear** → scalling, adding

$$T\{\alpha_1 x_1[n]\} = g[n]\,\alpha_1 x_1[n]$$

$$T\{\alpha_2 x_2[n]\} = g[n]\,\alpha_2 x_2[n]$$

$$T\{\alpha_1 x_1[n]\} + T\{\alpha_2 x_2[n]\} = g[n](\alpha_1 x_1[n] + \alpha_2 x_2[n])$$

↳ ①

Assume,

$\alpha_3 x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$

$\alpha_3 y_3[n] = \alpha_1 y_1[n] + \alpha_2 y_2[n]$

$$T\{\alpha_3 x_3[n]\} = g[n]\, \alpha_3 x_3[n]$$

$$\alpha_1 x_1[n] + \alpha_2 x_2[n] = g[n]\left[\alpha_1 x_1[n] + \alpha_2 x_2[n]\right] \quad — ②$$

$\hookrightarrow$ Eq (1) = Eq (2)

linear.

- **Time invariant.**

1) $T\{x[n-n_0]\} = g[n]\, x[n-n_0]$ — ① → delay of Input sequence

2) $y[n-n_0] = g[n-n_0]\, x[n-n_0]$ — ②

time shift

Eq (1) ≠ Eq (2)

⟹ Variant

- **Memory less**

الاعتماد على القيمة الحالية

$$T\{x[\underline{n}]\} = g[\underline{n}]\, x[\underline{n}]$$

Memory less → depends only on the n^{th} value of x.

2) $T\{x[n]\} = \sum_{K=n_0}^{n} x[K]$.

- **Stable:**

$$|T\{x[n]\}| = \left|\sum_{K=n_0}^{n} x[K]\right| = \sum_{K=n_0}^{n} \underbrace{|x[n]|} \rightarrow |x[n]| \leq M < \infty$$

$$\leq \sum_{K=n_0}^{n} M < \infty$$

$$\leq (n-n_0) M < \infty$$

↳ $n \Rightarrow$ finite. $\Rightarrow$ stable

↳ $n \rightarrow \infty \Rightarrow (n-n_0) M \rightarrow \infty$

↳ unstable.

٢) نقسم دي — يعتمد عليها الـ output سابقي

causal:

↳ non-causal.

١) نقسم دي يعتمد عليها الـ Input

linear: → Adding, scalling.

$$T\{\alpha_1 x_1[n]\} = \sum_{K=n_0}^{n} \alpha_1 x_1[K]$$

$$T\{\alpha_2 x_2[n]\} = \sum_{K=n_0}^{n} \alpha_2 x_2[K]$$

$$T\{\alpha_1 x_1[n] + \alpha_2 x_2[n]\} = \sum_{K=n_0}^{n} (\alpha_1 x_1[K] + \alpha_2 x_2[K]) \quad ①$$

Assume: $\alpha_3 x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$

$\alpha_3 y_3[n] = \alpha_1 y_1[n] + \alpha_2 y_2[n]$

$$T\{\alpha_3 x_3[n]\} = \sum_{K=n_0}^{n} \alpha_3 x_3[n]$$

$$T\{\alpha_1 x_1[n] + \alpha_2 x_2[n]\} = \sum_{K=n_0}^{n} (\alpha_1 x_1[n] + \alpha_2 x_2[n]) \to ②$$

$Eq(1) = Eq(2) \longrightarrow$ linear.

time_variant.

$$T(x[n]) = \sum_{K=n_0}^{n} x[K]$$

↳ delay of Input sequence:

حاطي بيه الاقواس للـ x بدري اعملنا shift

$$T\{x[n-n_0]\} = \sum_{K=n_0}^{n} x[K-n_0] \text{ —— } ①$$

↳ $u = K - n_0$

↳ $\sum_{u=0}^{n-n_0} x[u]$ —— ③

↳ delay time.

دين ماجي n اعملنا shift

$$T\{x[n-n_0]\} = \sum_{K=n_0}^{n-n_0} x[K] \text{ —— } ②$$

$\Rightarrow$ ~~time~~ $T\{x[n-n_0]\} \neq y[n-n_0]$

$\forall n - \{0\}$

Time_ variant.

- Memory less ← الاعتماد على الحاضر فقط.

↳ Memory. → لا تعتمد فقط على الـ present بسبب وجود أكثر من قيمة

$$③\ T\{x[n]\} = \sum_{k=n-n_0}^{n+n_0} x[k].$$

- stable:

$$|T\{x[n]\}| = \left|\sum_{k=n-n_0}^{n+n_0} x[k]\right| = \sum_{k=n-n_0}^{n+n_0} |x[k]| \Rightarrow |x[n]| \leq M < \infty$$

$\boxed{BI}$ ✓

$$\leq \sum_{k=\boxed{n-n_0}}^{n+n_0} M < \infty$$

constant ←

$$\leq (2n_0+\boxed{1})M < \infty$$

$n-n_0 - n-n_0$

$-2n_0 + \boxed{1}$ بسبب وجود الصفر

⊛ A

↳ stable

- causal:

↳ non-causal ⟶ depends on future value

$$\sum_{n-n_0}^{n+n_0}$$

$n+n_0$ → قيم مستقبلية.

$n-n_0$ → قيم ماضية

- linear: → Scalling, Adding.

↳ linear.

-

- **Time varient.**

↳ delay input sequence: $T\{x[n-n_0]\} = \sum_{k=n-n_0}^{n+n_0} x[k+n_0]$ — ①

↳ time shift: $y[n-n_0] = \sum_{k=(n-n_0)-n_0}^{(n-n_0)+n_0} x[k]$

$= \sum_{k=n-2n_0}^{n} x[k]$. — ②

↳ $u = k + n_0$

$\sum_{u=n-2n_0}^{n} x[u]$. — ③

Eq(1) = Eq(2)
↳ time invarient.

4) $T(x[n]) = x[n-n_0]$.

- **Stability**

$|T(x[n])| = |x[n-n_0]|$; $|x[n]| \leqslant M < \infty$

$\leqslant M < \infty$ ⟹ BIBO ⇒ stable.

- **causal**

↳ $T(x[n]) = x[n-n_0]$, $n_0 > 0$ ← causal
$= x[n+n_0]$ $n_0 < 0$ ← non-causal

- lineal

↳ linear system

- time varient.

delay Input sequence : $T\{x[n-n_0]\} = x[(n-n_0)-n_0]$ —①
time shift : $T\{x[n-n_0]\} = x[n-n_0-n_0]$ —②

Eq (1) = Eq (2) ⟶ time Invariant.

- Memory less

↳ Memory.

5) $T(x[n]) = e^{x[n]}$

- stability.

$|T\{x[n]\}| = e^{|x[n]|}$; $|x[n]| \leqslant M < \infty$ ⊛

$\leqslant e^{M} < \infty$ ⟶ stable.

linear:

$T\{\alpha_1 x_1[n]\} = e^{\alpha_1 x_1[n]}$
$T\{\alpha_2 x_2[n]\} = e^{\alpha_2 x_2[n]}$

$T\{\underbrace{\alpha_1 x_1[n] + \alpha_2 x_2[n]}_{=\alpha_3 x_3[n]}\} = \underline{e^{\alpha_1 x_1[n]} + e^{\alpha_2 x_2[n]}}$

↳ $\neq e^{\alpha_3 x_3[n]}$

⟶ non-linear.

- causal:

↳ causal:

- Memory less

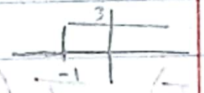
↳ Memory less

⑥ $T(x[n]) = x[n] + 3u[n+1]$.

- stability:

$$|T(x[n])| = |x[n] + 3u[n+1]|$$
$$\leq |x[n]| + |3u[n+1]|.$$

* $|x[n]| \leq M < \infty$



↳ $\begin{cases} M+3 & n > -1 \\ M+0 & n < -1 \end{cases} < \infty$ ↳ stable

BIBO

- causal:

↳ causal.

ما إلي دخل بـ u فقط x[n]

⊛ - linear:

$$\alpha_1 y_1[n] = \alpha_1 x_1[n] + 3\alpha_1 u[n+1]$$
$$\alpha_2 y_2[n] = \alpha_2 x_2[n] + 3\alpha_2 u[n+1]$$

$$\underline{\alpha_1 y_1[n] + \alpha_2 y_2[n]} = \alpha_1 x_1[n] + \alpha_2 x_2[n] + 3u[n+1][\alpha_1 + \alpha_2]$$

$$\alpha_3 y_3[n] = \underline{\alpha_3 x_3[n]} + 3\alpha_3 u[n+1]$$

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] + 3\alpha_3 u[n+1]$$

non-linear

Convolution :=)).

- Continues:

$x(t) \longrightarrow h(t) \longrightarrow y(t)$

- for LTI:

$y(t) = x(t) * h(t)$; where * convolution.

- Discrete

$x[n] \longrightarrow h[n] \longrightarrow y[n]$

for LTI:

$y[n] = x[n] * h[n]$

→ $$y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda.$$

OR.

$$y(t) = \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda.$$

→ $$y[n] = \sum_{K=-\infty}^{\infty} x[n-K]\, h[K].$$

For Impulse Response $x[t] = \delta(t)$ - continues

$x[n] = \delta[n]$ - Discrete

↳ continues:

$$y(t) = \int_{-\infty}^{\infty} \delta(\lambda)\, h(t-\lambda)\, d\lambda$$

$$= h(t)$$

↳ discrete:

$$y[n] = \sum_{K=-\infty}^{\infty} \delta[k]\, h[n-K]$$

convolution of Continues :=))

↳ By plotting

↳ By using Integral.

Conv. of Discrete

↳ By plotting

↳ By using Summation.

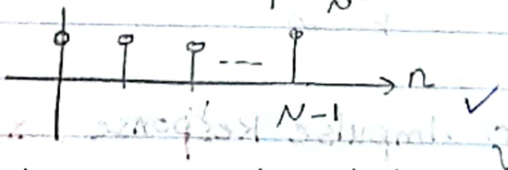
Example: Consider LTI system whith Impulse response
$$h[n] = u[n] - u[n-N],$$
and the Input Signal by $x[n] = a^n u[n]; 0<a<1$
evaluate $y[n]$

Ans: For LTI system :=))

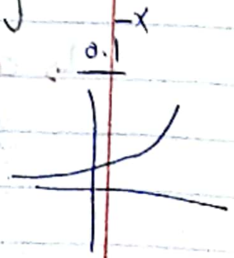
$y[n] = x[n] * h[n]$

where $h[n] = u[n] - u[n-N]$

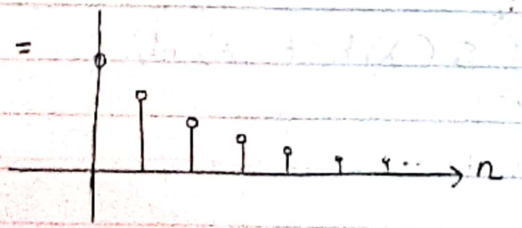
⊛ $h[n] =$

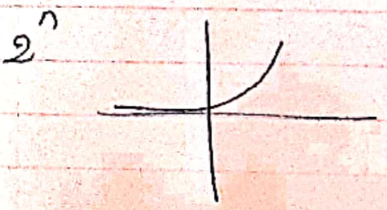


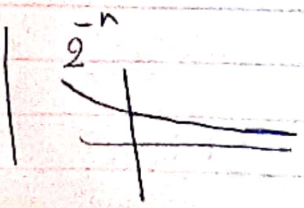
$$= \begin{cases} 1 & , 0 < n < N-1 \\ 0 & , o.w \end{cases}$$

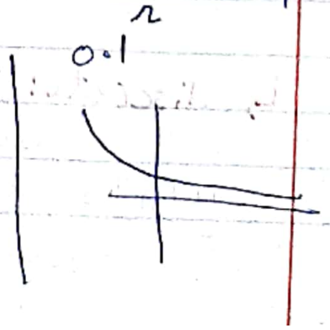


⊛ $x[n] =$









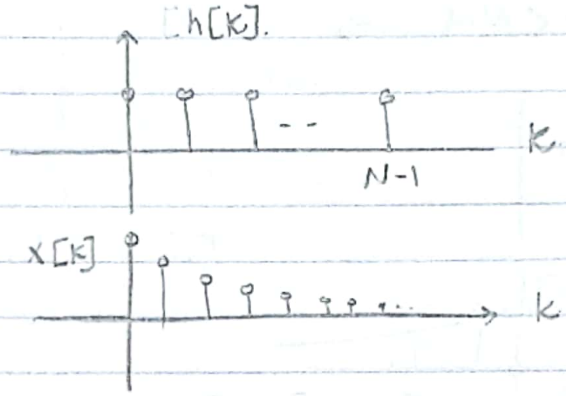
1) $h[n] \longrightarrow h[k]$

2) $x[n] \longrightarrow x[k]$.

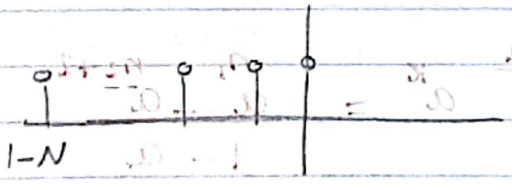
$$\Rightarrow \quad y[n] = \sum_{k=-\infty}^{\infty} x[k]\, \underline{h[n-k]}. \quad \text{OR}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

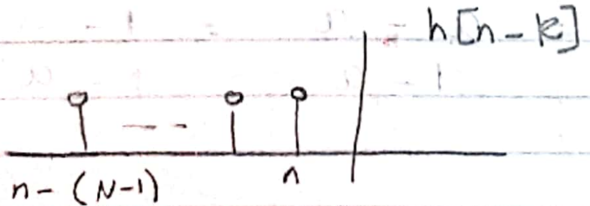
①



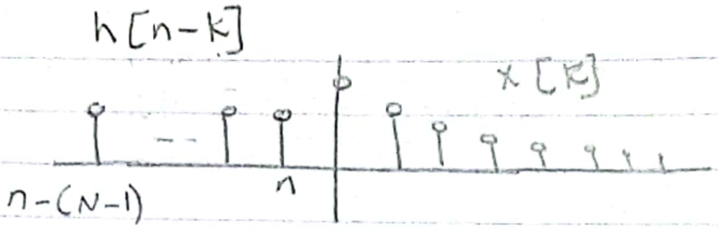
② $h[k] \rightarrow h[-k]$



3) Add n for $h[-k]$.



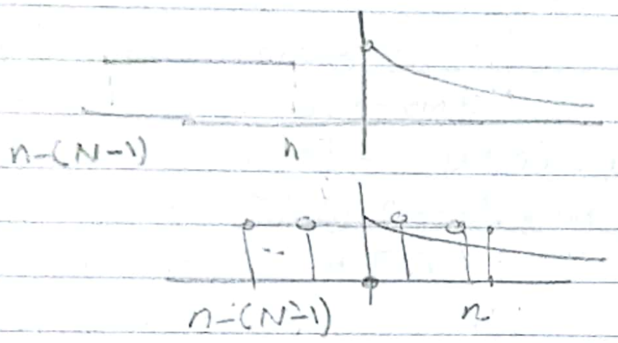
④ Apply convolution therorem



[0, N-1] , [0, ∞)
[0, N-1, ∞]

لايوجد تداخل بين الاقترانين ← - for $n < 0$ $\Rightarrow$ $\boxed{y[n] = 0}$

- for $0 \leqslant n < N-1$ $\Rightarrow$

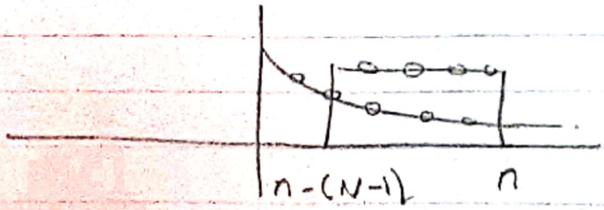


$$y[n] = \sum_{k=0}^{n} (1)(a^k)$$

In general: [power seriese] $\sum_{k=n_1}^{n_2} a^k = \frac{a^{n_1} - a^{n_2+1}}{1-a}$; $n_2 \geqslant n_1$

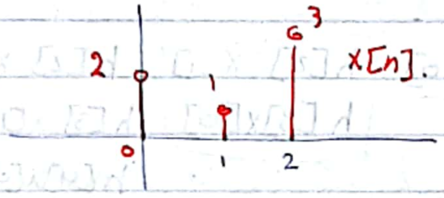
e.g. $\sum_{k=0}^{n} a^k = \frac{a^0 - a^{n+1}}{1-a} = \frac{1 - a^{n+1}}{1-a}$;

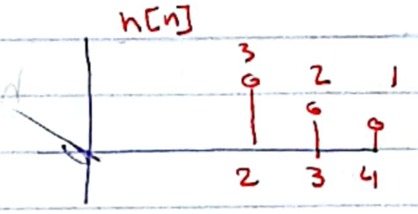
- for $n \geqslant N-1$: $\Rightarrow$ $n-1 \leqslant n < \infty$



$$y[n] = \sum_{k=n-(N-1)}^{n} a^{k} = \frac{a^{n-(N-1)} - a^{n}}{1-a}$$

Example: Consider LTI system, which $x[n]$ and $h[n]$ show below.

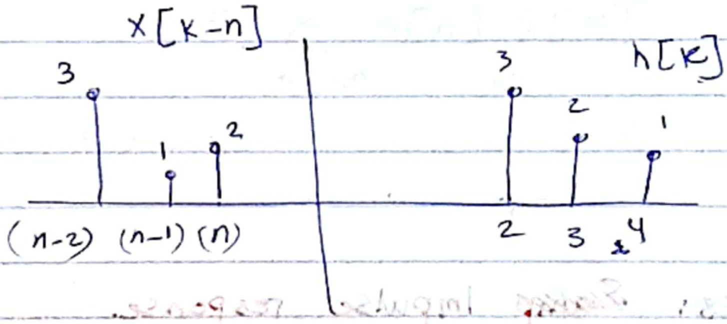




find $y[n]$

$[0,2]$, $[2,4] \Rightarrow [2,4,6]$

Ans:



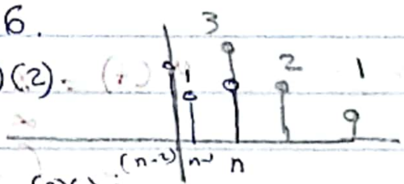
Method 1:

for $n < 2 \Rightarrow y[n] = 0$.

for $n = 2 \Rightarrow y[2] = (3)(2) = 6$.

for $n = 3 \Rightarrow y[3] = (3)(1) + (2)(2)$.

for $n = 4$ $\qquad = 7$



$\Rightarrow y[4] = (2)(1) + (1)(2) + (3)(3)$

$= 2 + 2 + 9 = 13$.

for $n = 5 \Rightarrow y[5] = (1)(1) + (3)(2) = 7$

for $n = 6 \Rightarrow y[6] = (3)(1) = 3$.

for $n = 7 \Rightarrow$ Zero.

بضل ماشيه طولما اصل لجواب صفر.

Method 2: - finite system:

n:	0	1	2	3	4	
x[n]:	x[0]	x[1]	x[2]			
h[n]:			h[2]	h[3]	h[4]	
			h[2] x[0]	$\boxed{\begin{matrix} h[2]\ x[1] \\ h[3]x[0] \end{matrix}}$	h[2] x[2] h[3] x[1] h[4]x[0]	h[3]x[2] h[4] x[1]	h[4]x[2]
$\sum$			(3)(2) 6	(3)(1) + (2)(2) 13	7	3	

- **Continues:** [illegible] Impulse response.

↳ ~~x[n]~~ x(t) = δ(t)

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda.$$

$$= \int_{-\infty}^{\infty} \delta(t-\lambda)\, h(\lambda)\, d\lambda.$$

$$= h(t)$$

In general: In Impulse respons $\rightarrow x(t) = \delta(t)$
$\hookrightarrow y(t) = h(t)$
$\hookrightarrow$ for $t > 0$.

e.g. find the impulse response for the following system

$$Rc \frac{dy(t)}{dt} + y(t) = x(t)$$

* when $x[n] = \delta[n]$

$$\Rightarrow y[n] = \sum_{k=-\infty}^{\infty} \delta[k] h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[n] \delta[k].$$ "step function".

$\Rightarrow$ when $k > 0$

Example: determine the Impulse response of the following system

$$y[n] = a_1 x[n] + a_2 x[n-1] + a_3 x[n-2] + a_4 x[n-3]$$

for Impulse response $\rightarrow x[n] = \delta[n]$
$\hookrightarrow y[n] = h[n]$

$\Rightarrow$ To find the Impulse response

$$h[n] = a_1 \overset{\delta[n-0]}{\delta[n]} + a_2 \delta[n-1] + a_3 \delta[n-2] + a_4 \delta[n-3]$$

$$= \{ \overset{n=0}{a_1}, \overset{n=1}{a_2}, \overset{n=2}{a_3}, \overset{n=3}{a_4} \}$$

نط ↘ **Example:** Determine the Impulse response of the discrete time accumulator.

$$y[n] = \sum_{L=-\infty}^{\infty} x[L]$$

the Impulse response of the system :

*
لازم نفهم
الدنيا أكبر
من صفر
ولا اقل من
صفر بعضهم
بجاوب هم

$$\Rightarrow h[n] = \sum_{L=-\infty}^{\textcircled{n}} \delta[n] = u[n]$$

$$= \sum_{L=0}^{\infty} \delta[n]$$

Example: An LTI system has the Impulse response $h[n] = \{\overset{-1}{1}, \overset{0}{\underline{2}}, \overset{1}{0}, \overset{2}{-3}\}$, and the Input sequence expresed as:

① $x_1[n] = \delta[n]$

② $x_2[n] = \delta[n+1] + \delta[n-2]$

③ $x_3[n] = \{\underline{1}, 1, 1\}$

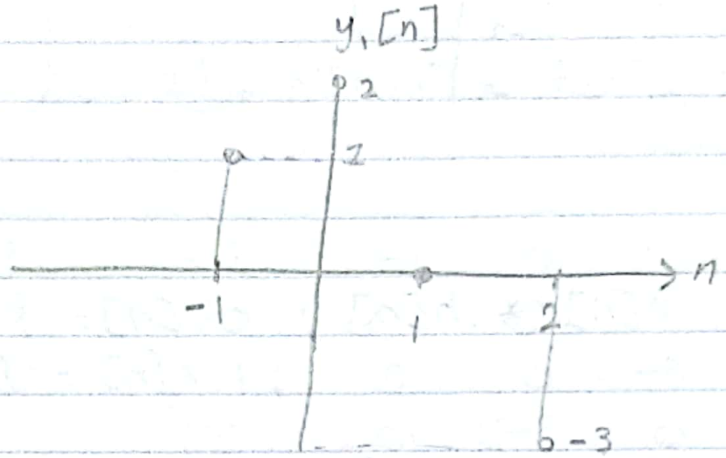
④ $x_4[n] = \{2, 1, \underline{-1}, -2, -3\}$

① find the output sequence $y[n]$

② plot the output sequence.

① for LTI system: $y[n] = x[n] * h[n]$.

$x_1[n] = \delta[n] \Rightarrow y_1[n] = x_1[n] * h[n]$
$= \delta[n] * h[n]$.
$= h[n]$.
$= \{1, \underline{2}, 0, -3\}$



② $x_2[n] = \delta[n+1] + \delta[n-2]$.

عليه ال delay حاي في input لازم اشوفه في output.

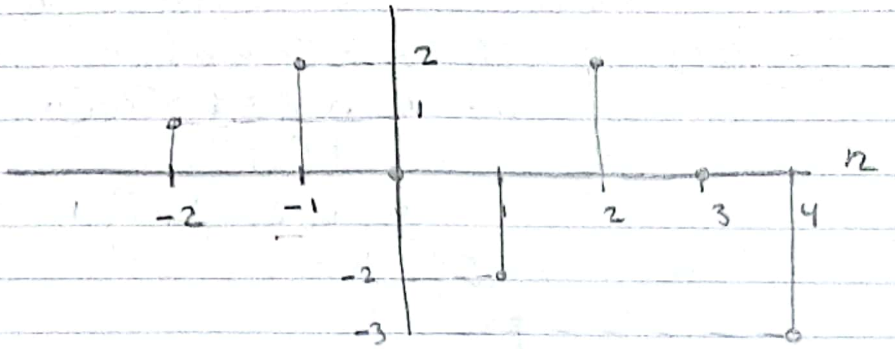
$y_2[n] = x_2[n] * h[n]$
$= (\delta[n+1] + \delta[n-2]) * h[n]$

$= \delta[n+1] * h[n] + \delta[n-2] * h[n]$.
$= h[n+1] + h[n-2]$

	n:	-2	-1	0	1	2	3	4
	$h[n]$:	0	1	2	0	-3	0	0
[shift left]	$h[n+1]$:	1	2	0	-3	0	0	0
[= right]	$h[n-2]$:	0	0	0	1	2	0	-3
		1	2	0	-2	2	0	-3

+

$y_2[n] = \{1, 2, \underline{0}, -2, 2, 0, -3\}$



③ $y_3[n] = x[n] * h[n]$; $h[n] = \{1, \underline{2}, 0, -3\}$

; $x_3[n] = \{\underline{1}, 1, 1\}$

$$X_3[n] = \{\underline{1}, 1, 1\} = \delta[n] + \delta[n-1] + \delta[n-2]$$

$$y_3[n] = x_3[n] * h[n]$$
$$= (\delta[n] + \delta[n-1] + \delta[n-2]) * h[n]$$
$$= \delta[n] * h[n] + \delta[n-1] * h[n] + \delta[n-2] * h[n]$$
$$= h[n] + h[n-1] + h[n-2].$$

n:	-1	<u>0</u>	1	2		
h[n]:	1	2	0	-3		
h[n-1]:	0	1	2	0	-3	
h[n-2]:	0	0	1	2	0	-3
Σ	1	<u>3</u>	3	-1	-3	-3

$$y_3[n] = \{1, \underline{3}, 3, -1, -3, -3\}$$

طريقة (2)

h[n] / x3[n]	1	2	0	-3
1	1	2	0	-3
1	1	2	0	-3
1	1	2	0	-3

$\Rightarrow y[n] = \{1, 3, 3, -1, -3, -3\}$.

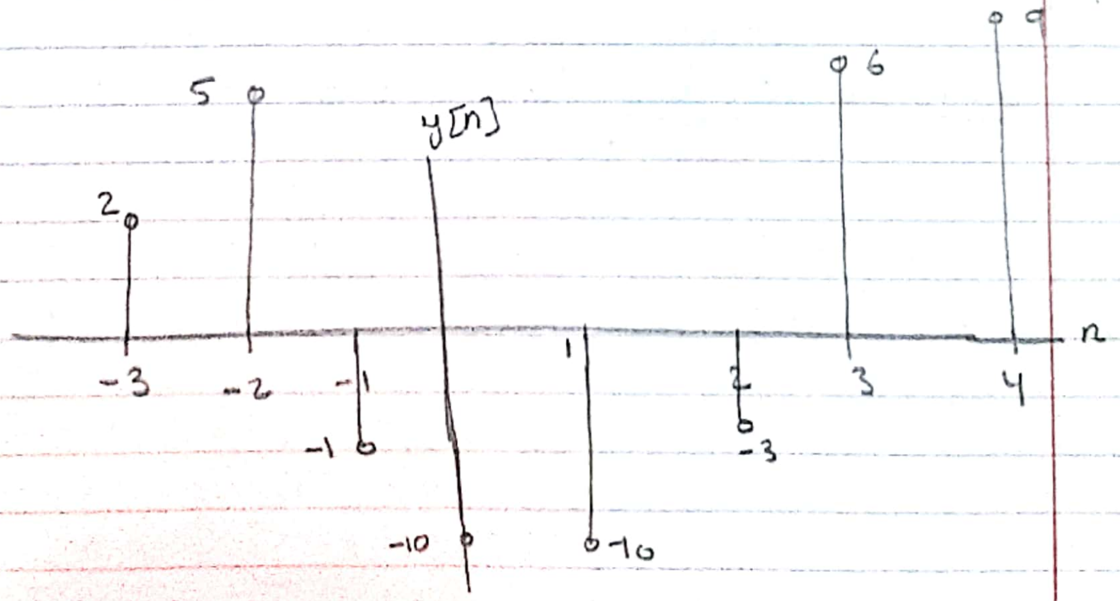
④ $y_4[n] = x_4[n] * h[n]$. ; $x_4[n] = \{2, 1, \underline{-1}, -2, -3\}$

$x_4[n] = 2\delta[n+2] + \delta[n+1] - \delta[n] - 2\delta[n-1] - 3\delta[n-2]$

$y_4[n] = [2\delta[n+2] + \delta[n+1] - \delta[n] - 2\delta[n-1] - 3\delta[n-2] * h[n]$

$= 2h[h+2] + h[n+1] - h[n] - 2h[n-1] - 3h[n-2]$

n:	-3	-2	-1	0	1	2	3	4
$-h[n]$:	0	0	$-(1)$	$-(2)$	0	$-(-3)$	0	0
$h[n+1]$	0	1	2	0	-3	0	0	0
$(2)h[n+2]$	$2(1)$	$2(2)$	$2(0)$	$2(-3)$	0	0	0	0
$(-2)h[n-1]$	0	0	0	$-2(1)$	$-2(2)$	$-2(0)$	$-2(-3)$	0
$(-3)\ h[n-2]$	0	0	0	0	$-3(1)$	$-3(2)$	$-3(0)$	$-3(-3)$
	2	5	$\boxed{-1}$	-10	-10	-3	6	9



$\boxed{\text{Example 3 =))}}$

$h[n] = \{-1, \underline{2}, 0, 1\}$

$x[n] = \{3, 1, \underline{0}, -1\}$

find $y[n]$:=))

طريقة ثانيه

لإيجاد $y[n]$

$\boxed{n = 0}$

$h[n]$ \ $y[n]$	-1	2	0	1
3	-3	6	0	3
1	-1	2	0	1
$\boxed{n=0} \rightarrow 0$	0	$\boxed{0}$	0	0
-1	1	-2	0	-1

$$y[n] = \{\underset{-3}{-3}, \underset{-2}{5}, \underset{-1}{2}, \underset{0}{\boxed{4}}, -1, 0, -1\}$$

properties of LTI system discrete

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n].$$

for LTI system ⇒ $y[n] = x[n] * h[n]$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[h-k]$$

$$= \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

causality for LTI system :

$$* \quad y[\underline{n_o}] = \sum_{k=-\infty}^{\infty} x[\underline{k}]\, \underbrace{h[n_o-k]}_{(1)} \rightarrow \text{if } n_o \geq k \Rightarrow \text{causal}$$

otherwise ⇒ non causal

دائماً في causality لأي نظام بيكون ضبط الرقم إلي ب output أكبر أو يساوي الرقم الموجود في ال Input.

Impulse Response:

↳ $x[n] = \delta[n]$ and $\underbrace{h[n] = 0}_{(2)} \Rightarrow \boxed{n < 0}$

for (1) and (2)

$n = n_o - k \Rightarrow k = n_o - n \Rightarrow n_o \geq n_o - n \Rightarrow n \geq 0$

stability for LTI system:

↳ $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \right|$$

$$\leq \sum_{k=-\infty}^{\infty} |x[k]|\, |h[n-k]| \quad ; \text{ since } \Rightarrow$$

↳ since $|x[k]| \leq M < \infty \Rightarrow$ BI

$$\hookrightarrow \sum_{k=-\infty}^{\infty} |\underbrace{h[n-k]}_{finite}| < \infty$$

- Passive and lossless system

↳ A discrete time system is defined to be passive if, for energy finite Input $x[n]$, the output $y[n]$ has at most the same energy

$$\hookrightarrow |y[n]|^2 \leq \sum_{k=-\infty}^{\infty} |x[n]|^2$$

← قيمة ال E يلي بنحصل عليها أقل أو تساوي قيمة ال E يلي جاي من ال Input لأنه قيمة ال E يلي جاي من (Passive circuit) يلي عندي تقريبا تساوي واحد. فالتأثير الحقيقي من قيمة ال Input.

↳ for a lossless system, the above inequality is satisfied with an equal sign for every point.

$$\hookrightarrow |y[n]|^2 = \sum_{k=-\infty}^{\infty} |x[n]|^2$$

Example: consider the discrete time system, defined by $y[n] = \alpha x[n-N]$ with N a positive integer, check if:

1) the system is passive,
2) " " " lossless.

Ans:

↳ The output energy of the system can be given by

$$|y[n]|^2 = |\alpha x[n-N]|^2$$

$$\leq |\alpha|^2 |x[n-N]|^2 \quad ; \quad |\alpha|^2 < 1$$

↳ passive

عشان أضمن
إن ال E
اللي ب out
أقل من ال
E اللي
ب Input
[passive]

$$\Rightarrow |\alpha| = 1 \Rightarrow \text{lossless}$$

Example 8=)) the Impulse response for the following system

$$y[n] = a_1 x[n] + a_2 x[n-1] + a_3 x[n-2] + a_4 x[n-3]$$

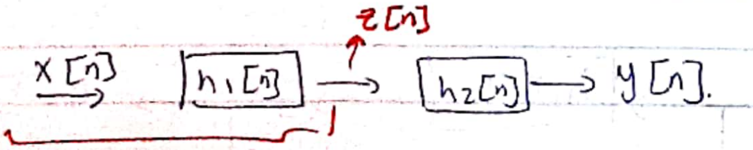
Ans:

$$h[n] = a_1 \delta[n] + a_2 \delta[n-1] + a_3 \delta[n-2] + a_4 \delta[n-3]$$

$$= \{\underline{a_1}, a_2, a_3, a_4\}$$

Interconnection schemes of LTI system

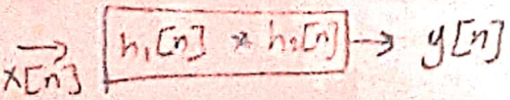
↳ cascade connection



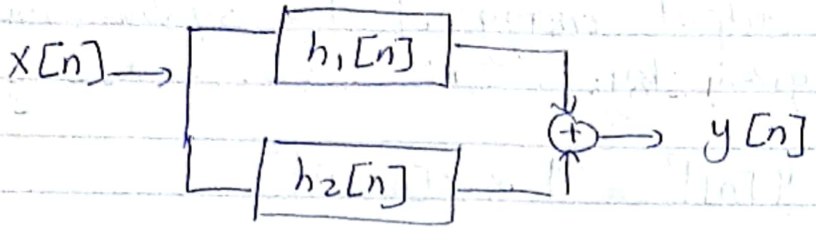
↳ $z[n] = x[n] * h_1[n]$

↳ $y[n] = h_2[n] * z[n]$

↳ $y[n] = x[n] * h_1[n] * h_2[n]$

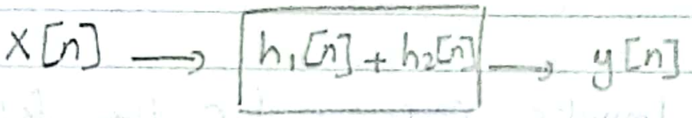


↳ parallel connection



↳ $y[n] = x[n] * (h_1[n] + h_2[n])$

$= x[n] * h_1[n] + x[n] * h_2[n]$



Example: Consider a discrete time system, where

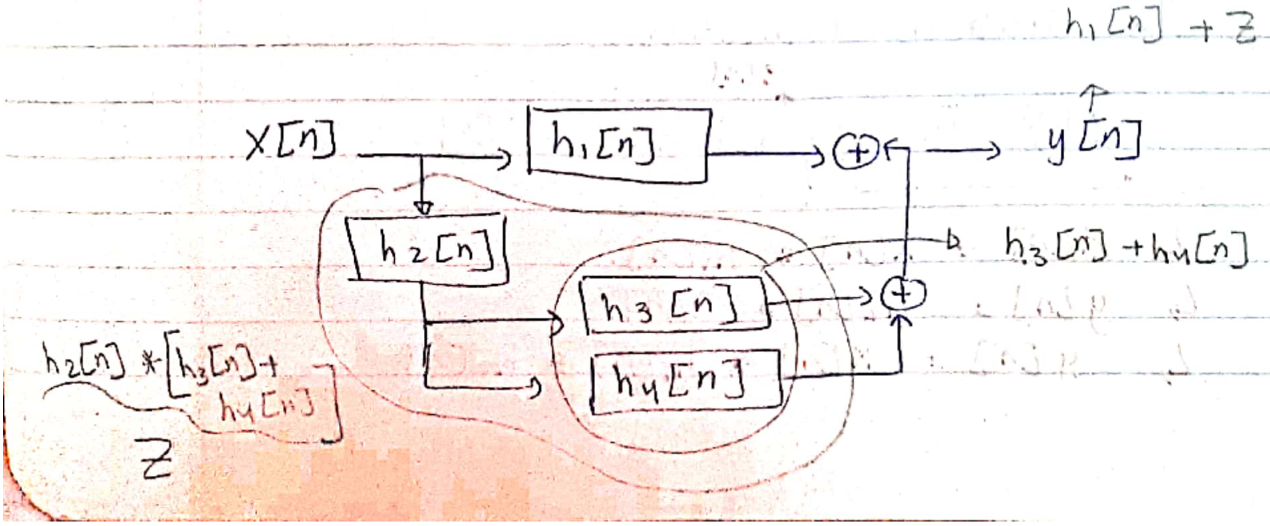
$h_1[n] = \delta[n] + \frac{1}{2}\delta[n-1]$

$h_2[n] = \frac{1}{2}\delta[n] + -\frac{1}{4}\delta[n-1]$

$h_3[n] = 2\delta[n]$

$h_4[n] = -2\left(\frac{1}{2}\right)^n u[n]$

Evaluate the overall impulse response h[n].



Ans8=)

$$x[n] \longrightarrow \boxed{h_1[n] + h_2[n] * (h_3[n] + h_4[n]} \longrightarrow y[n]$$

$$h[n] = h_1[n] + h_2[n] * [h_3[n] + h_4[n]]$$

$$h[n] = h_1[n] + h_2[n] * h_3[n] + \underline{h_2[n] * h_4[n]}$$

where : $h_2[n] * h_4[n]$

$$= \left(\frac{1}{2} \delta[n] - \frac{1}{4} \delta[n-1]\right) * \left(-2 \left(\frac{1}{2}\right)^n u[n]\right)$$

$$= \left(\frac{1}{\cancel{2}}\right)(-\cancel{2})\left(\frac{1}{2}\right)^n u[n] \, \delta[n] +$$

$$-\left(\frac{1}{\cancel{4}_2}\right) \delta[n-1] \cdot (-\cancel{2}) \left(\frac{1}{2}\right)^n u[n]$$

$$= -\left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{(n-1)} u[n-1]$$

$$= -\left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^n u[n-1]$$

$$= -\left(\frac{1}{2}\right)^n \left[\underline{u[n] - u[n-1]}\right]$$

$$= -\left(\frac{1}{2}\right)^n [\delta[n])$$ ∵ "<u>sampling theorem</u>"

$$= -\left(\frac{1}{2}\right)^0 \delta[n]$$

$$= - \delta[n]$$

$$x[n] * \delta[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n-k]$$

$\delta[n-1]\ \delta[n] =$
$\delta[n-1-0]$
$\delta[n-1]$

$\Rightarrow h_2[n] * h_3[n]$

$$\left(\frac{1}{2}\delta[n] - \frac{1}{4}\delta[n-1]\right) * 2\delta[n]$$

$$\delta[n] - \frac{1}{2}\delta[n-1]$$

$$h[n] = h_1[n] + h_2[n] * h_3[n] + h_2[n] * h_4[n]$$

$$= \left(\cancel{\delta[n]} + \cancel{\frac{1}{2}\delta[n-1]}\right) + \left(\delta[n] - \cancel{\frac{1}{2}\delta[n-1]}\right) - \cancel{\left(\delta[n]\right)}$$

$$= \delta[n]$$

$$\xrightarrow{x[n]} \boxed{\delta[n]} \longrightarrow y[n]$$

"buffer"
"regester" → خزن بيانا
بعرها بوقتها
"memory"

$$y[n] = \delta[n] * x[n].$$

In general: $y[n] = x[n] * \delta[n-n_0]$ "convolution theorem"
$= x[n-n_0]$

$y[n] = x[n]\ \delta[n-n_0]$ Sampling theorem
$= x[n_0]\ \delta[n-n_0]$

Example: Consider the discrete system with an Impulse response. determine the value of $h_2[n]$.

$$x[n] \rightarrow \boxed{h[n] = u[n]} \rightarrow \boxed{\text{Black Box}} \rightarrow y[n] = x[n]$$

$$x[n] \rightarrow \boxed{h[n] = \delta[n]} \rightarrow y[n] \quad \hookrightarrow y[n] = \delta[n]$$

$\hookrightarrow \; h_1[n] * h_2[n] = \delta[n]$

$u[n] * h_2[n] = \delta[n]$

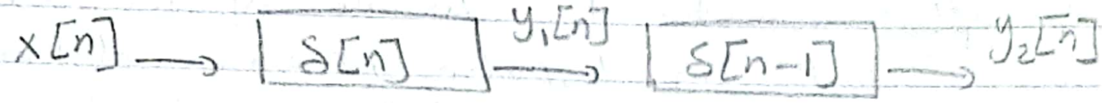
In general: $u[n] - u[n-1] = \delta[n]$

$\hookrightarrow \; h_2[n] = \delta[n] - \delta[n-1]$

$$u[n] * (\delta[n] - \delta[n-1]) =$$
$$= u[n] * \delta[n] - u[n] * \delta[n-1]$$
$$= u[n] - u[n-1]$$
$$= \delta[n]$$

$\hookrightarrow$ Inverse $u[n] = u[n] - u[n-1]$
$= \delta[n]$

(*)
دائماً لما اعمل
كونفوليوشن بين
two system
وكان بنتج
$\delta[n]$ ، فانا
لازم اكون مدركه
انو واحد من ال system
هو ال Inverse ل
ال system الثاني.



$$y_2[n] = x[n] * [\delta[n] * \delta[n-1]]$$
$$= x[n] * \delta[n-1]$$
$$= x[n-1]$$

⁛ Correlation of Discrete time system:=))

- **auto correlation:=))** $R_{xx}[K] = \sum_{m=-\infty}^{\infty} x[m]\, x[m-K]$

آلية المقارنة بين صورتين متشابهان

نركز كانه عندي اشاره مخزنه واشارة متأخرة عنها بلحظة معينة ونشوف اشوف مقدار التطابق بين طول الإشارتين

$$= X \cdot X$$

$$= |X||X| \cos\phi$$

$$\hookrightarrow R_{xx}[0] = |X|^2 = \sum_{m=-\infty} |X|^2$$

- **Cross - correlation:**

آلية المقارنة بين صورتين مختلفات

$$R_{xy}[K] = \sum_{m=-\infty}^{\infty} x[m]\, Y[m-k]$$

* **Properities :=))**

- $R_{xx}[K] = R_{xx}[-K]$ even function

- $R_{xx}[K] = \sum_{m=-\infty}^{\infty} x[m]\, x[\underset{\sim}{m-k}] \longrightarrow *$

$$= \sum_{m=-\infty}^{\infty} x[m]\, x[-(k-m)]$$

$$= x[k] * x[-k]$$

الاختلاف الوحيد هو

$x[m-k]$

$h[n-m]$

"اشارة"

In general: the convolution theorem

$$y[n] = x[n] * h[n]$$

$$= \sum_{m=-\infty}^{\infty} x[m]\, h[\underset{\sim}{n-m}] \longrightarrow *$$

$\hookrightarrow$ continues: $\int x(\lambda)\, h(\underset{\sim}{t-\lambda})\, d\lambda$

Corrolation Coefficient ;=))

$$C_{xy}[k] = \frac{R_{xy}[k]}{\sqrt{R_{xx}[k]\, R_{yy}[k]}}$$

"Maximum value, when k=0"

$$C_{xy} = \frac{R_{xy}[0]}{\sqrt{R_{xx}[0]\; R_{yy}[0]}} = \frac{|X|\,|Y|\cos\phi}{\sqrt{|x|^2\, |y|^2}} = \cos(0) = 1$$

في هاي الحالة بكونوا ماخذين نفس الاشارة.

$(C_{xy} = 1)$ OR $C_{xy} = -1$ OR $C_{xy} = 0$

اشارتين متساويات في المقدار، مختلفات في الاتجاه.

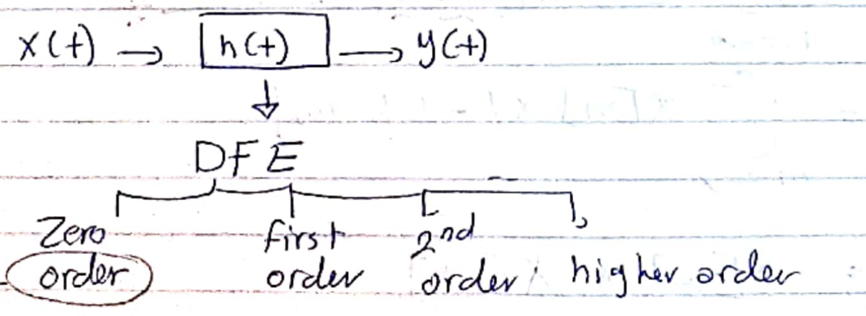
اشارتين مختلفات

↳ $-1 \leq C_{xy} \leq 1$

↳ $R_{xx}[k] = x[k] * x[-k]$

↳ $R_{xy}[k] = x[k] * y[-k]$

Continues:



يتم قياسة من خلال قيمة اعلى مشتقة في ال output

↳ $y(t) = x(t)$ Zero order

↳ $\frac{dy(t)}{dt} + y(t)$ 1^{st} order

← اعلى درجه لـ summation بالنسبة للـ output ، صغيرا = صفة الـ order

Discrete:

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$$

↓ DFE

$\boxed{\text{Order} = N}$

$$\sum_{K=0}^{N} a_K \, a[y-K] = \sum_{m=0}^{M} b_m x[n-m]$$

أهم اشي الـ output ←

$$\hookrightarrow \text{zero order: } \; a_0 y[n] = \sum_{m=0}^{M} b_m x[n-m]$$

الـ Input ما بيهمني ← معنى الـ order

$$\hookrightarrow a_0 y[n] + a_1 y[n-1] = \sum_{m=0}^{M} b_m [n-M]$$ 1st order

← بشوف اعلى رقم في منطقة الـ shift بالنسبة للـ output

In generel:

$$\sum_{K=0}^{N} a_K y[n-K] = \sum_{m=0}^{M} x[n-m]$$

$$a_0 y[n] + \sum_{K=1}^{N} a_K y[n-K] = \sum_{m=0}^{M} b_m x[n-m] \text{ —— } *$$

In general: in convolution therom

$$y[n] = \sum_{K=-\infty}^{\infty} h[K] \, x[n-k] \text{ —— } *$$

if we assume ⇒ $a_0 = 1$, and $a_K = 0$ ؛ $K = 1, 2, \ldots N$

$$\hookrightarrow y[n] = \sum_{m=0}^{m} b_m \, x[n-m]$$

$$\Rightarrow h[n] = \begin{cases} b_n , & n = 0, 1, \dots M \\ 0 , & 0.w \end{cases}$$

Example: determine the Impulse response of the first order system

$$y[n] = x[n] + a y[n-1]$$

Assume $h[n] = 0 , \quad n < 0$

Example "Correlation"

①

$- x[n] = \{-1, 2, 1\}$

$x[-n]$ \ $x[n]$	$\overset{0}{-1}$	2	1
1	−1	2	1
2	−2	4	2
0 −1	①	−2	−1

$\boxed{n=0}$

$$P_{xx}(L) = \frac{\{-1, 0, \underset{\uparrow}{6}, 0, -1\}}{\{6\}}$$

$$P_{xx}(L) = \frac{r_{xx}(L)}{r_{xx}[0]}$$

$$\underline{P_{xx}(L)} = \{-0.166, \overset{-1}{0}, \overset{\downarrow_0}{1}, 0, -0.166\}$$

↳ $P_{xx}(0) = 1$

↳ $P_{xx}(-1) = 0$

② $x[n] = \{①, 1, 2, 2\}$ $y[n] = \{①, 2, 3, 4\}$

↳ find $r_{xy}[L]$, $P_{xy}[L]$

$y[-n]$ \ $x[n]$	1	1	2	2
4	4	4	8	8
3	3	3	6	6
2	2	2	4	4
1	1	1	2	2

$$r_{xy} = \{4, 7, 13, \underset{\uparrow}{17}, 11, 6, 2\}$$

$r_{xx}[0] = 10$

$r_{yy}[0) = 30$

	1	1	2	2
2	2	2	4	4
2	2	2	4	4
1	1	1	2	2
1	1	1	2	2

$$r_{xx} = \{2, 4, 7, \underset{\uparrow}{10}, 7, 4, 2\}$$

$\boxed{n=0}$

	1	2	3	4
4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4

$$r_{yy} = \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$$

$n=0$

$$\rho_{xy}[l] = \frac{\{4, 7, 13, 17, 11, 6, 2\}}{\sqrt{(10)(30)}} = \frac{--}{17.32}$$

Example 3:

find r_{xy}

r_{xz}

ρ_{xy}

ρ_{xz}

$x[n] = \{1, 3, -2, 4\}$

$y[n] = \{2, +3, -1, 3\}$

$z[n] = \{-2, 4, -1, 2\}$

→ r_{xy}:

$= \{3, 8, -6, \underset{\uparrow}{25}, -4, 8, 8\}$

$r_{xx}[0] = 30$

$r_{yy}[0] = 23$

	1	3	-2	4
3	3	9	-6	12
-1	-1	-3	2	-4
3	3	9	-6	12
2	2	6	-4	~~5~~8

→ r_{xz}:

$= \{2, 5, 3, 20, -18, 20, 8\}$

$r_{xx}[0] = 30$

$r_{zz}[0] = 25$

	1	3	-2	4
2	2	6	-4	8
-1	-1	-3	2	-4
4	4	12	-8	16
-2	-2	-6	4	-8

$$\rho_{xz} = \frac{\{2, 5, 3, 20, -18, 20, 8\}}{\sqrt{30 \times 25}}$$

$$\rho_{xy} = \frac{\{3, 8, -6, 25, -4, 8, 8\}}{\sqrt{30 * 23}}$$

Fourier Transform for DT.

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

- fourier transform for DT :

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]\, e^{-jwn}$$ → The frequency response of the system

↳ w: continues variable

↳ n: discrete "

$$\int_{-\pi}^{\pi} H(e^{jw})\, \underline{e^{jwm}} = \int_{-\pi}^{\pi} \sum_{n=-\infty}^{\infty} h[n]\, e^{-jwn}\, \underline{e^{jwm}}$$

بكامل ←

بدلالة .w

$$H(e^{jw}) \equiv H(e^{jw+2\pi k})$$

k: int

$$\boxed{\text{discret}} = \int_{-\pi}^{\pi} H(e^{jw})\, e^{-jwm}\, dw = \sum_{n=-\infty}^{\infty} h[n] \int_{-\pi}^{\pi} e^{-jw(n-m)} \cdot dw$$

for F $n \neq m \Rightarrow h[n]$ undefined.

for $n = m \Rightarrow$

$$\hookrightarrow \int_{-\pi}^{\pi} H(e^{jw})\, e^{-jwn}\, dw = 2\pi\, h[n]$$

$$\left[\text{continues}\right] \Rightarrow h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{jw})\, e^{jwn}\, dw.$$

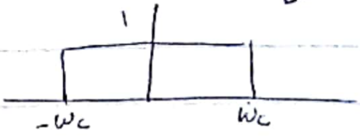
Inverse fourier transform

Type of filters :

$$H_L(e^{jw}) = \begin{cases} 1, & |w| \leq w_c \\ 0, & \text{o.w} \end{cases}$$

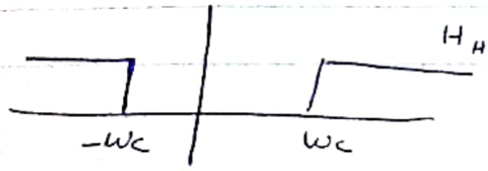
$w < |w| < 2\pi$

1. low pass filter



$H_{HP}(e^{jw})$

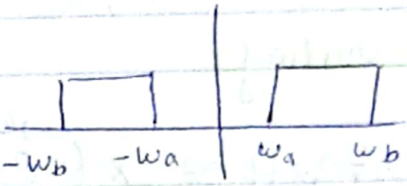
2. High pass filter.



In general:

$$H_{PF}(e^{jw}) = 1 - H_{LP}(e^{jw})$$

3. Band pass filter:



Example: evaluate the frequency response of low pass filter if the fourer frequency of LPF can be given by:

$$H_{LP}(e^{jw}) = \begin{cases} 1 & |w| \leqslant w_c \\ 0 & w_c < |w| \leqslant 2\pi \end{cases}$$

Ans:

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{jw}) e^{jwn} dw$$

$$= \frac{1}{2\pi} \int_{-w_c}^{w_c} 1 \, e^{jw_n} dw.$$

$$= \frac{1}{j \cdot n(2\pi)} \left(e^{jw_c n} - e^{-jw_c n} \right) = \frac{\sin(nw_c)}{\pi n}$$

دائماً Low pass filter ⇒ non-causal system

Fourier transform for DT therom:

1- **Linearity.** $x[n] \rightarrow \boxed{LTI} \rightarrow y[n]$

$$F\left[a x_1[n] + b x_2[n]\right] = a X_1(e^{jw}) + b X_2(e^{jw})$$

2. **Time shifting.**

$$F[x[n \mp n_d]] = X(e^{jw})\, e^{\mp jwn_d}.$$

- frequency shifting $F[x[n]\, e^{+jw_0 n}] = X(e^{j(w - w_0)})$

3. **Time reversal**

$$x[n] \xrightarrow{F} X(e^{jw})$$
$$x[-n] \longrightarrow X(e^{-jw}) \equiv X^*(e^{jw})$$

if $x[n]$ real

4. **diffrential in frequency**

$$n\, x[n] \xrightarrow{F} \boxed{j}\, \frac{dX(e^{jw})}{dw}$$

5. **Parseval's therom.**

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \underbrace{\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw}$$

$|X(e^{jw})|^2 =$ Energy spectral density.

where $|X(e^{jw})|^2$: energy density spectrum (EDS)

6. Convolution therom.

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$ $\quad y[n] = x[n] * h[n$

$$F[x[n] * h[n]] = \underline{X(e^{jw})\ H(e^{jw})}\quad Y(e^{jw})$$

$$x[n] * h[n] \xleftrightarrow{F} X(e^{jw})\,H(e^{jw}) \qquad \boxed{\text{duality}}$$

7. Modulation and windowing therom

$$x[n]\, w[n] \xleftrightarrow{F} X(e^{jw}) \circledast W(e^{jw})$$

e.g: $x[n]\, e^{jw_o n} \xrightarrow{F} X(e^{j(w-w_o)})$

Example: Consider the following system

$$y[n] = x[n] \cos(w_o n)$$

evaluate $Y(e^{jw})$

Ans:

$$Y(e^{jw}) = F[x[n] \cos(w_o n)]$$

$$= F\left[\frac{1}{2} x[n]\, e^{jw_o n} + \frac{1}{2} x[n]\, e^{-jw_o n}\right]$$

linearity $$= F\left[\frac{1}{2} x[n]\, e^{jw_o n}\right] + F\left[\frac{1}{2} x[n]\, e^{-jw_o n}\right]$$

$$= \frac{1}{2} X[e^{j(w-w_o}] + \frac{1}{2} X(e^{j(w+w_o)})$$

$$\cancel{\tfrac{1}{2}}\, x[n]\left[\frac{e^{jw_n} + e^{-jwn}}{2}\right] = \frac{1}{2} x[n]\, e^{jw_n} + \frac{1}{2} x[n]\, e^{-jwn}$$

طريقه (2)

$$F\left[\frac{1}{2}x[n]\,e^{j\omega_0 n}\right] \equiv \frac{1}{2}F\left[x[n]\underbrace{e^{j\omega_0 n}}_{w[n]}\right]$$

$$= \frac{1}{2}F\left[w[n]\,x[n]\right]$$

$$= \frac{1}{2}F\left[x[n]\right] * F\left[w[n]\right]$$

$$= \frac{1}{2}X(e^{j\omega}) * F\left[w[n]\right]$$

where $F[w[n]] = F[e^{j\omega_0 n}]$

$$F[\delta[n-nd]] = \sum_{n=-\infty}^{\infty} \delta[n-nd]\,e^{-j\omega n}$$

$$= e^{-j\omega nd}$$ [sampling theorem]

$$\to \frac{1}{2}X\left(e^{j(\omega-\omega_0)}\right)$$

Example: Consider the following system

$$x[n] \rightarrow \overset{\text{LTI}}{\boxed{h[n]}} \rightarrow y[n]$$

if $x[n] = \delta[n-nd]$, find $Y(e^{j\omega})$
$\boxed{\text{shift}}$

$$y[n] = x[n] * h[n]$$

$$= \delta[n-nd] * h[n]$$

$$= h[n-nd].$$

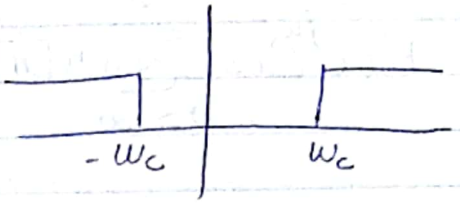
$$Y(e^{j\omega}) = F[h[n-nd]] = H(e^{j\omega})\,e^{-j\omega nd}$$

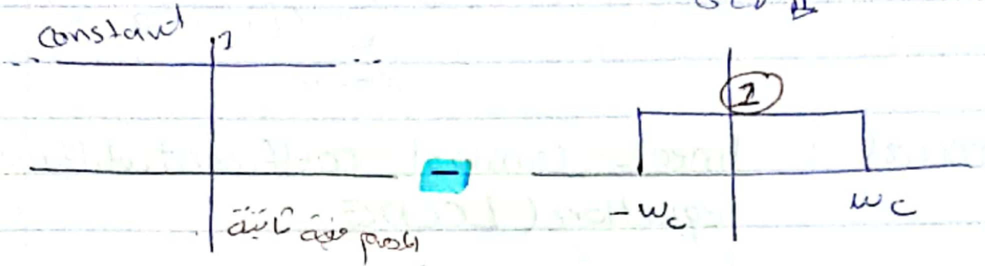
تم التغير كله ال phase

(✳) **Example:** evaluate the frequency response of HPF. /with delay $z^{-j\omega n}$

Ans: $e^{j(0)}$ $e^{j(\omega nd)}$

$H_{LP} = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & o.w \end{cases}$





$H_{HP}(e^{j\omega})$ idial $= (1 - H_{LP}(e^{j\omega})) = \delta[n] - \frac{\sin(\omega_c n)}{\pi n}$

(✳) $\left[h_{HP}(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{HP}(e^{j\omega}) e^{j\omega n} d\omega \right.$

$H_{HP} = [1 - H_{LP}(e^{j\omega})] \cdot (1)(e^{-j\omega nd}) = e^{-j\omega nd} - H_{LP}(e)^{j\omega nd}$ ✱

$= \delta[n-nd] - ~~\sin~~($

$- \frac{\sin(\omega_c(n-n}{\pi(n-nd)}$

Example: $y[n] = x[n] + ay[n-1]$; $H(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$ ✓

$F\{y[n] - ay[n-1]\} = F\{x[n]\}$

$Y(e^{j\omega}) - aY(e^{j\omega})e^{-j\omega} = X(e^{j\omega})$

$\rightarrow \boxed{H(e^{j\omega}) = Y(e^{j\omega}) / X(e^{j\omega}).}$

$Y(e^{j\omega})[1 - ae^{-j\omega}] = X(e^{j\omega})$

$\boxed{H(e^{j\omega})} = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 - ae^{-j\omega}}$ #

↓ Frequency Response. ✓

$h[n] = a^n u[n]$

تكملة ⇒

L13:

LTI system

LTI system.

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

⇒ for LTI system: $y[n] = x[n] * h[n]$

$$= \sum_{K=-\infty}^{\infty} x[n-K]\, h[n]$$

$$= \sum_{K=-\infty}^{\infty} x[K]\, h[n-K]$$

مهم cr's
في لدرسيه

in general: linear constant coefficient diffrence equation (LCCDE

N^{th} order ←

$$\sum_{K=-\infty}^{N} a_K\, y[n-K] = \sum_{m=-\infty}^{M} b_m\, x[n-m]$$

e.g : Zero order:

$$a_0 y[n] = \sum_{m=0}^{M} b_m\, x[n-m] \quad \text{if } a_0 = 1$$

$$\Rightarrow y[n] = \sum_{m=0}^{M} b_m\, x[n-m] \Rightarrow$$

$$h[n] = \begin{cases} b_n & , n = 0, 1, \ldots, M \\ 0 & , \text{o.w} \end{cases}$$

:

$u[3-u]$

$u[-u+3]$

Example: consider the following differences equation : system $y[n] = x[n] + a\,y[n-1]$, evaluate the response of the system if $x[n] = K\delta[n]$ and $y[-1] = c$

Ans:=>

for $\boxed{n \geq 0}$ [Forward]

$$y[0] = K\delta[0] + a\,y[-1]$$
$$= (K)(1) + ac$$
$$= K + ac$$

$$y[1] = K\delta[1] + a\,y[0]$$
$$= 0 + a[K + ac]$$
$$= aK + a^2c$$

$$y[2] = K\delta[2] + a\,y[1]$$
$$= 0 + a(aK + a^2c)$$
$$= a^2K + a^3c$$

$$\Rightarrow y[n] = [a^nK + a^{n+1}c]\,u[n].$$

للقانون نعوض $n \geq 0$ ✓

$\boxed{\text{for } n \leq -1}$ reverse. $y[n-1] = (y[n] - x[n])\,a^{-1}$

$$y[n-1] = a^{-1}[y[n] - x[n]]$$

$$y[-2] = a^{-1}[y[-1] - \underbrace{K\delta[-1]}_{\text{zero}}]$$
$$= a^{-1}C$$

$$y[-3] = a^{-1}(y[-2] - 0) = a^{-2}c$$

$$y[n] = a^{n+1}C.$$

$$\Rightarrow \quad y[n] = \underbrace{a^{n+1} C}_{\text{مشتركة بين الحالتين}} + \underline{a^{n} K\, u[n]}.$$

[ثابت] عشان هيك مابضربه بـ u[n]

عشان اضمن انه هاد الجزء بيبدأ منطقة الاكبر من صفر

Frequency Response.

$$x[n] = e^{j w_0 n}$$

LTI

$$\longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = x[n] * h[n] = \sum_{K=-\infty}^{\infty} x[n-K]\, h[K]$$

$$= \sum_{K=-\infty}^{\infty} e^{j w_0 (n-K)}\, h[K] \quad ✓$$

Input.

$$= e^{j w_0 n} \left[\sum_{K=-\infty}^{\infty} e^{-j w_0 K}\, h[K] \right] \quad ✓$$

$$\hookrightarrow \boxed{H(e^{j w_0 K})}$$

$$\Rightarrow \text{In general:} \quad H(e^{j w_0 n}) = \sum_{n=-\infty}^{\infty} e^{-j w_0 n}\, h[n] \quad ✓$$

where :

$$\text{complex \#} \multimap H(e^{j w_0 n}) = |(e^{j w_0 n})| \angle \Phi_{H(e^{j w_0 n})}$$

$$\hookrightarrow |H e^{j w}|\, e^{j \Phi_{H(e^{j w})}}$$

In general:

$$|H(e^{j w_0 n})| = |H(e^{-j w_0 n})| \Rightarrow \text{even}$$

$$\angle \Phi_{H(e^{j w_0 n})} = - \angle \Phi_{H(e^{-j w_0 n})} \Rightarrow \text{odd}$$

Example: Consider the ideal delay system defined by: $y[n] = x[n-n_d]$

* find the frequency response $H(e^{j\omega_0 k})$ of the system

Ans:

$$e^{j\omega_0 n}\; \underset{\sim}{x[n]} \longrightarrow \boxed{h[n]} \longrightarrow \underset{\sim}{y[n]}.$$

$$y[n] = x[n-n_d]$$

$$= e^{j\omega_0 (n-n_d)} = e^{j\omega_0 n}\, \underbrace{e^{-j\omega_0 n_d}}_{H(e^{j\omega_0 n})}$$

↳ since $y[n] = e^{j\omega_0 n} \, H(e^{j\omega_0 k})$

$$\equiv e^{j\omega_0 n} \sum_{k=-\infty}^{\infty} e^{-j\omega_0 k} \, h[k]$$

$$\Rightarrow H(e^{j\omega_0 k}) = e^{-j\omega_0 n_d}$$

$$= \underbrace{|H(e^{j\omega_0 k})|}_{\text{magnitude}} \, \underbrace{e^{j\Phi_H(e^{j\omega_0 k})}}_{\text{phase}}$$

$|H(e^{j\omega_0 k})| = 1$ and

$$\Phi_H(e^{j\omega_0 n}) = -\omega_0 n_d$$

Example: Evaluate the response of the system if the Input signal is defined as:

$$x[n] = A\cos(w_0 n + \Phi)$$

Ans:

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

$$= \sum_{k=-\infty}^{\infty} A\cos(w_0(n-k)+\Phi)\, h[k]$$

↳ By using Eulars' equation:

$$A\cos(w_0(n-k)+\Phi) = \frac{A}{2} e^{j(w_0(n-k)+\Phi)} + \frac{A}{2} e^{-j(w_0(n-k)+\Phi)}$$

$$\Rightarrow y[n] = \sum_{k=-\infty}^{\infty} \frac{A}{2} e^{j(w_0(n-k)+\Phi)} \cdot h[k] + \sum_{k=-\infty}^{\infty} \frac{A}{2} e^{-j(w_0(n-k)+\Phi)} h[k]$$

$$= \sum_{k=-\infty}^{\infty} \frac{A}{2} e^{jw_0 n} e^{-jw_0 k} e^{j\Phi} h[k] + \sum_{k=-\infty}^{\infty} \frac{A}{2} e^{-jw_0 n} e^{jw_0 k} e^{-j\Phi} \cdot h[k]$$

$$= \frac{A}{2} e^{jw_0 n} e^{j\Phi} \underbrace{\sum_{k=-\infty}^{\infty} h[k]\, e^{-jw_0 k}} + \frac{A}{2} e^{-jw_0 n} e^{-j\Phi} \underbrace{\sum_{k=-\infty}^{\infty} e^{jw_0 n} h[k]}$$

$$H(e^{jw_0 k}) \Rightarrow |H(e^{jw_0 k})|\, e^{j\Phi_{H(e^{jw_0 k})}} \qquad H(e^{-jw_0 k})$$

- In general: $H(e^{jw_0 k}) = |H(e^{jw_0 k})| \angle \Phi_{H(e^{jw_0 k})}$

$$|H(e^{-jw_0 k})|\, e^{-j\Phi_{H(e^{-jw_0 k})}}$$

- $H(e^{-jw_0 k}) = |H(e^{-jw_0 k})| \angle \Phi_{H(e^{-jw_0 k})}$

$$\Rightarrow \left|H(e^{j\omega_0 k})\right| = \left|H(e^{-j\omega_0 k})\right|$$ even function

$$\Rightarrow \angle\Phi_{H(e^{j\omega_0 k})} = -\angle\Phi_{H(e^{-j\omega_0 k})}$$ odd "

$$\Rightarrow y[n] = \frac{A}{2} e^{j\omega_0 n} e^{j\Phi} \left|H(e^{j\omega_0 k})\right| e^{j\Phi_{H(e^{j\omega_0 n})}} +$$
$$\frac{A}{2} e^{-j\omega_0 n} e^{-j\Phi} \left|H(e^{j\omega_0 k})\right| e^{-j\Phi_{H(e^{j\omega_0 k})}}$$

$$= \frac{A}{2}\left|H(e^{j\omega_0 k})\right| \left[e^{j(\omega_0 n + \Phi + \Phi_{H(e^{j\omega_0 k})})} + e^{-j(\omega_0 n + \Phi + \Phi_{H(e^{j\omega_0 k})})} \right]$$

$$= A\,\underbrace{\left|H(e^{j\omega_0 k})\right|}\ \cos\left(\omega_0 n + \Phi \mp \underbrace{\Phi_{H(e^{j\omega_0 k})}}\right)\Big]$$

Example: Consider the difference equation defined by $y[n] - ay[n-1] = x[n]$

ⓐ find the $\boxed{\text{Impulse response}} \xrightarrow{\text{يعني}} \underset{\sim}{n \geq 0}$

$$y[n] = x[n] + a\,y[n-1]$$

for Impulse Response: $x[n] = \delta[n]$
$y[n] = h[n]$, $n \geq 0$

always

$$\Rightarrow h[n] = a^n u[n]$$

* freq. Response is periodic with period 2π:

$$H(e^{j(\omega_0 + 2\pi)}) = \sum_{k=-\infty}^{\infty} h[k]\, e^{-j(\omega_0 + 2\pi)n} = e^{-j2\pi n} \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\omega_0}$$

imp ↘

(b) find the frequency response. $H(e^{jw_0k}]$

Ans:

$$H(e^{jw_0}) = \sum_{K=-\infty}^{\infty} e^{-jw_0k}\, h[K]$$

$$= \sum_{K=-\infty}^{\infty} e^{-jw_0k}\, a^k u[k]$$

$$= \sum_{K=0}^{\infty} \left(a e^{-jw_0}\right)^k$$

$$\text{Since } \sum_{K=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \quad ; \boxed{0<\alpha<1}$$

$$H(e^{jw_0}) = \frac{1}{1 - a e^{-jw_0}}$$

$$= \frac{1}{1 - a[\cos(w_0) - j\sin(w_0)]}$$

$$= \frac{1}{(1 - a\cos(w_0) + aj\sin(w_0))}$$

$$= \frac{1\angle 0}{\sqrt{(1-a\cos(w_0))^2 + (a\sin(w_0))]^2} \angle \tan^{-1}\left(\frac{a\sin w_0}{1-a\cos(w_0)}\right)}$$

$$= \frac{1\angle 0}{\sqrt{1 - 2a\cos(w_0) + a^2\cos^2(w_0) + a^2\sin^2(w_0)}} \angle 1$$

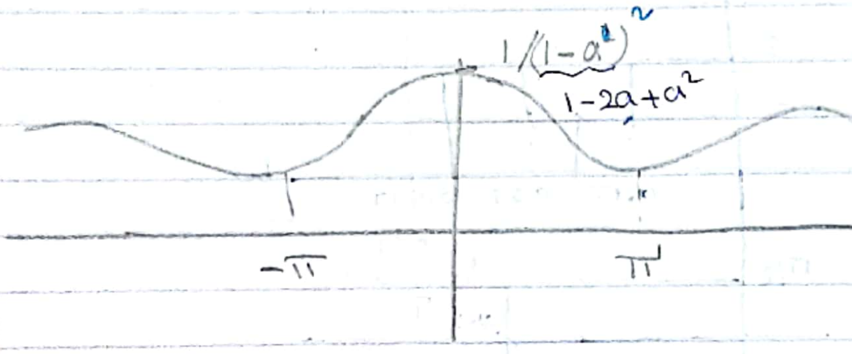
$$\tan^{-1}\left(\frac{a\sin w_0}{1-a\cos w_0}\right)$$

$$|H(e^{jw_0})| = \frac{1}{\sqrt{1+a^2-2a\cos(w_0)}}$$

$$\angle \Phi_{H(e^{jw_0})}' = -\tan^{-1}\left(\frac{a\sin(w_0)}{1-a\cos(w_0)}\right)$$

↳ Sketch $|H(e^{jw_0})|^2$

نرسم من أول صفرة ⇒ $$|H(e^{jw_0})|^2 = \frac{1}{1-2a\cos(w_0)+a^2}$$



⇒ output discrete if input periodic signal

input: non-periodic ⇒ continuous signal

example: $h[n] = \delta[n-nd]$, evaluate the frequency Response.

Ans:

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]\, e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} \delta[n-nd]\, e^{-jwn}$$

$$= e^{-jw\,nd}$$

magnitude: 1

phase: $-wnd$

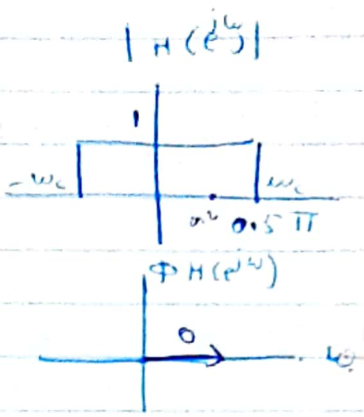
example:

$X[n] : 5 \cos(0.2\pi)$

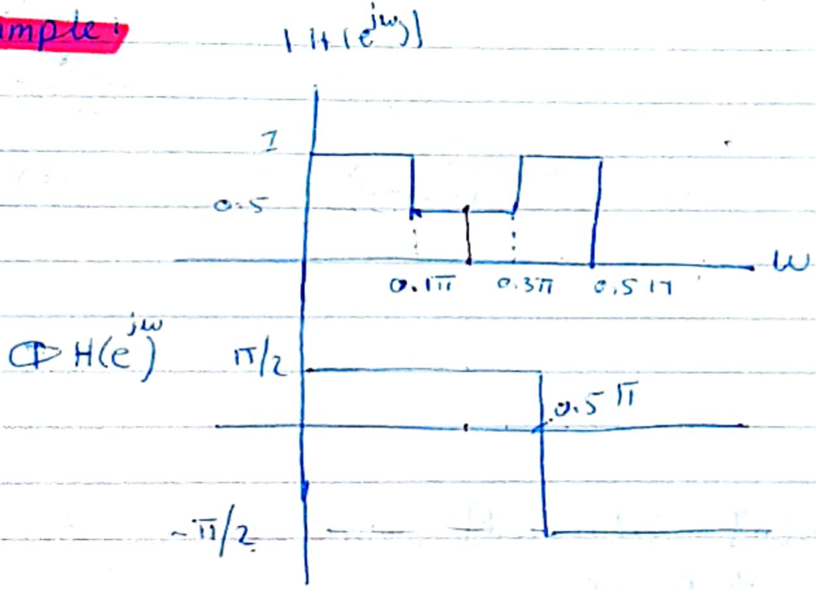
$X_2[n] : 5 \cos(0.8\pi)$

$\hookrightarrow y_1[n] = (1) 5 \cos(0.2\pi + 0))$

$\hookrightarrow y_2[n] = 0$



example:



$x_1[n] = 5 \cos(\underbrace{0.2}\pi n)$

$\hookrightarrow y_1[n] = \underbrace{\left(\frac{1}{2}\right)} 5 \cos(0.2\pi n + \pi/2)$

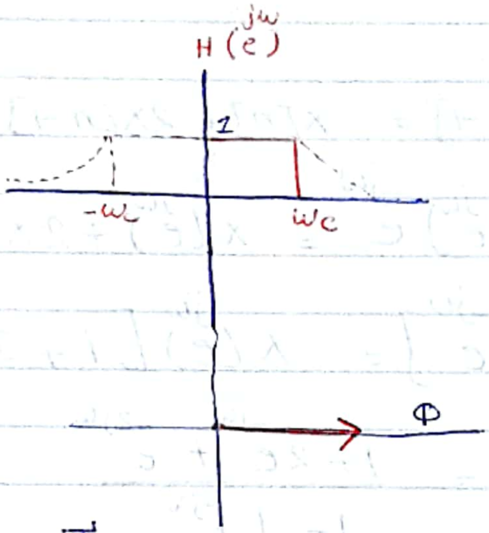
$x_2[n] = 8 \cos(0.4\pi n)$

$\hookrightarrow y_2[n] = (1) 8 \cos(0.4\pi n + \frac{\pi}{2})$

$x_3[n] = 10\cos(0.8\pi n)$

↳ $y_3[n] =$ zero

* **Example:**

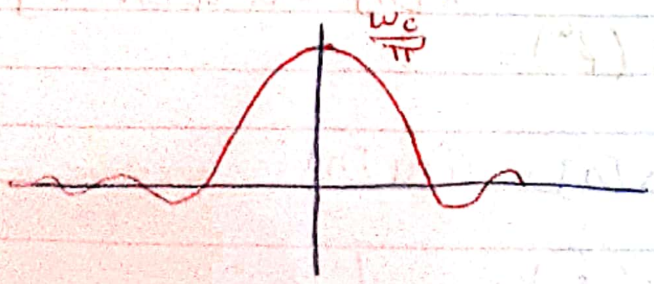


$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} (1)\, e^{j w_n}\, dw$$

$$= \frac{1}{2\pi} \cdot \frac{e^{jwn}}{jn}\Big|_{-w_c}^{w_c}$$

$$= \frac{1}{2\pi} \cdot \frac{1}{jn}\left[\frac{e^{jw_c n} - e^{-jw_c n}}{2j}\right]$$

$$= \frac{\sin(w_c n)}{\pi n} \cdot \frac{w_c}{w_c} = \frac{\operatorname{sinc}(w_c n)}{\pi} \cdot w_c$$



⇒ ~~pro~~ Fourier transform for DT theorem

Example :=) $y[n] - \frac{1}{2} y[n-1] = x[n] + 2x[n-1] + x[n-2]$

evaluate $H(e^{j\omega})$. freq. Response

$$F[\, y[n] - \frac{1}{2} y[n-1] = x[n] + 2x[n-1] + x[n-2] \,]$$

$$= Y(e^{j\omega}) - \frac{1}{2} Y(e^{j\omega}) e^{-j\omega} = X(e^{j\omega}) + 2X(e^{j\omega}) e^{-j\omega} + X(e^{j\omega}) e^{-j2\omega}$$

$$= Y(e^{j\omega}) \left[1 - \frac{1}{2} e^{-j\omega}\right] = X(e^{j\omega}) \left[1 + 2e^{-j\omega} + e^{-2j\omega}\right]$$

$$= H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 + 2e^{-j\omega} + e^{-2j\omega}}{1 - \frac{1}{2} e^{-j\omega}}$$

$$= \frac{1}{1 - \frac{1}{2} e^{-j\omega}} + \frac{2e^{\boxed{-j\omega}}}{1 - \frac{1}{2} e^{-j\omega}} + \frac{e^{\boxed{-2j\omega}}}{1 - \frac{1}{2} e^{-j\omega}}$$

$$= \left(\frac{1}{2}\right)^n u[n] + 2\left(\frac{1}{2}\right)^{\boxed{n-1}} u[\underline{\underline{n-1}}] + \left(\frac{1}{2}\right)^{\boxed{n-2}} u[\underline{\underline{n-2}}]$$

Example: determine the fourier transform of the system.

$$x[n] = a^n u[n-5]$$

$$X(e^{j\omega}) = ?$$

$$\sum_{n=-\infty}^{\infty} a^n u[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(a e^{-j\omega}\right)^n$$

$$\left[\; x[n] = a^n u[n] \qquad X(e^{j\omega}) = \frac{1}{1 - a e^{-j\omega}} \right.$$

$a^n u[n-5]$ —— لازم يكونو (نفس ال shift)

$x[n] = 5(1)^n u[n]$

~~$X(Z) = 5\delta[n]$~~

$\sum 5 z^{-n}$

$\frac{5}{1-z^{-1}}$

$\frac{1}{1-ae^{jw}}$

$$x[n] = a^n u[n-5] \frac{a^{-5}}{a^{-5}}$$

$$= \left(a^{n-5} u[n-5]\right) \cdot \frac{1}{a^{-5}}$$

$$= a^5 \left[a^{n-5} u[n-5]\right]$$

$$= a^5 \left[\frac{1}{1-ae^{-jw}}\right] e^{-jw(5)}$$

Example :=))

Inverse fourier transform :

$$x[e^{jw}] = \frac{1}{(1-ae^{-jw})(1-be^{-jw})} \quad , \text{ evaluate } x[n]$$

Ans:=))

$$\frac{1}{(1-ae^{-jw})(1-be^{-jw})} = \frac{A}{1-ae^{-jw}} + \frac{B}{1-be^{-jw}}$$

$$= \frac{A(1-be^{-jw}) + B(1-ae^{-jw})}{(1-ae^{-jw})(1-be^{-jw})}$$

$$1-be^{-jw} = 0 \Rightarrow be^{-jw} = 1$$

$$e^{-jw} = \frac{1}{b} \quad , \quad e^{-jw} = \frac{1}{a}$$

$$\Rightarrow 1 = A\overset{0}{\cancel{(1-b(\tfrac{1}{b})}} + B\left(1 - a\left(\tfrac{1}{b}\right)\right)$$

$\boxed{9:30 \rightarrow 6}$

$$\Rightarrow B\left(1-\frac{a}{b}\right)=1$$

$$B\left(\frac{b-a}{b}\right)=1 \Rightarrow \boxed{B=\frac{b}{b-a}}$$

$$1= A\left(1-b\left(\frac{1}{a}\right)\right)+B\left(1-\cancel{a\left(\frac{1}{a}\right)}\right)$$

$$1 = A\left(1-\frac{b}{a}\right)$$

$$\Rightarrow \boxed{A=\frac{a}{a-b}}$$

$$\Rightarrow x[n] = A\,a^n u[n] + B\,b^n u[n]$$

$$x[n]=\left(\frac{a}{a-b}\right)a^n u[n]+\left(\frac{b}{b-a}\right)b^n u[n] \quad \#.$$

Example: evaluate the FT of the signal

$$x[n]=(n+1)a^n u[n]$$

Ans: $x[n] = \underbrace{n\,a^n u[n]}+a^n u[n]$

$$* \; F[n\,x[n]] = j\,\frac{d\,X(e^{j\omega})}{d\omega}$$

$$\hookrightarrow j\frac{d}{d\omega}\left[\frac{1}{1-a e^{-j\omega}}\right]+\frac{1}{1-a e^{-j\omega}}$$

$$=\frac{1}{(1-a e^{-j\omega})^2}.$$

L14:=11

Fourier transform for DT system

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} \longrightarrow \omega: \text{ continues variable}$$

↳ n: discrete variable

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega.$$

Example: Consider the following system with Complex response

i.e

$$h[n] = \left(\frac{j}{2}\right)^n u[n]$$

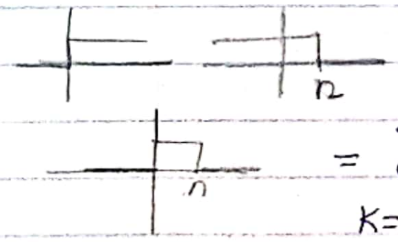
with input $x[n] = \cos(n\pi)\,u[n]$, find $y[n]$

Assume LTI

For LTI $\Rightarrow y[n] = h[n] * x[n]$

u[K]. u[n-K]

$$= \sum_{K=-\infty}^{\infty} h[K]\, x[n-K]$$



$$= \sum_{K=-\infty}^{\infty} \left(\frac{j}{2}\right)^K u[K] \cdot \cos((n-K)\pi)\, \cancel{u[n]}\, u[n-K]$$

$$= \sum_{K=0}^{N} \left(\frac{j}{2}\right)^K \cos((n-K)\pi) = \underline{(-1)^n} \left[\frac{1-(-j/2)^{n+1}}{1+j/2}\right]$$

cos ل بسبب وجود

لانه مرة (+) ومرة (-)

Example: determine the input response of the high pass filter with delay:

$$H_{HP}(e^{j\omega}) = \begin{cases} e^{-j\omega nd} & , \omega_c < |\omega| < \pi \\ 0 & , |\omega| < \omega_c \end{cases}$$

$$H_{HP}(e^{j\omega}) = 1 - H_{LP}(e^{j\omega})$$

$$H_{HP}(e^{j\omega}) = e^{-j\omega nd}\left(1 - H_{LP}(e^{j\omega})\right)$$

$$h\cancel{H}_{HP}[n] = \delta[n-nd] - h_{LP}(n-nd)$$

$$h_{HP}[n] = \delta[n-nd] - \frac{\sin(\omega_c(n-nd))}{\pi(n-nd)}$$

$x[n] = 2^n u[n]$ unstable

$x[n] = 2^n u[n] \cdot \left(\frac{1}{r}\right)^n$. stable; $r > 2$

Z- transform 8=))

In Continues 8=))

DFE: time - domain → freq. domain → S. domain

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$
↳ DFE

In Discrete 8=))

n- domain → freq. domain → $\boxed{\text{Z- transform [ROC]}}$
↳ Region of conv

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-jwn}$$

⇒ In general: forier transform:

$$\hookrightarrow X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jwn}.$$

$; Z = e^{jw}. \Rightarrow |z|\, e^{j\Phi_z}$ [magnitude + phase].

$$\hookrightarrow X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

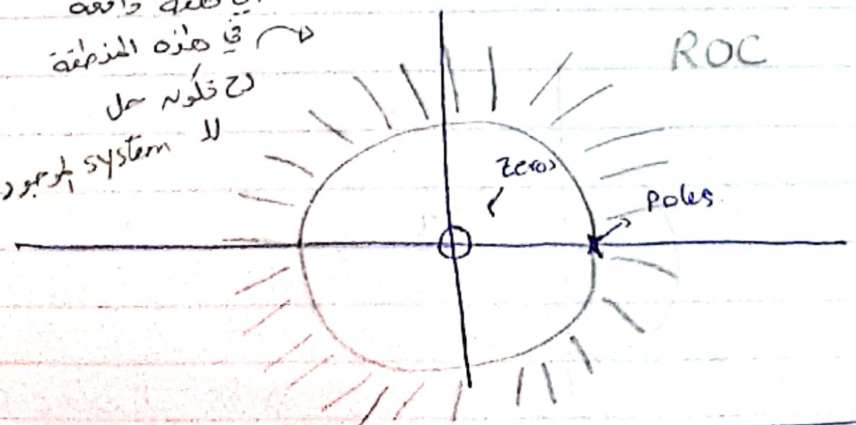
$$X(z) = \frac{(z - z_1)(z - z_2)}{(p - p_1)(p - p_2)}$$

Zero's O

poles X

أي نقطة واقعة في هذه المنطقة رح تكون حل لل system الموجود

منطقة الحل بيتم ايجادها من جذور ال **Poles**



e.g: $X(z) = \frac{z}{z-1}$

و يلزم أصفار المقام أكبر من صفر

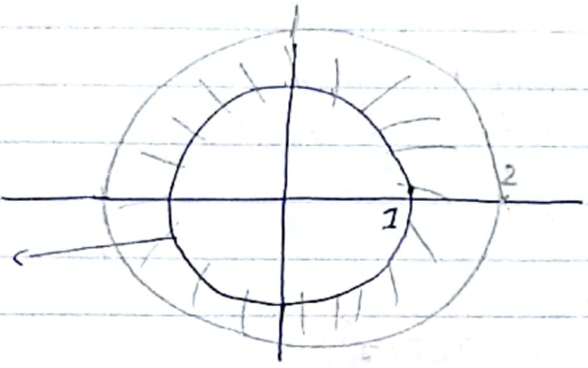
* to find ROC $\boxed{z - 1 > 0} \Rightarrow \boxed{|z| > 1}$

* if ROC $\rightarrow \infty$ $\Rightarrow$ Right-sided exponential sequence.

e.g. 2: $X(Z) = \frac{1}{2 - Z}$

To find ROC $\Rightarrow$ $2 - Z > 0$ $\Rightarrow$ $|Z| < 2$.

FT exist.



$\Rightarrow$ Left sided exponential sequence

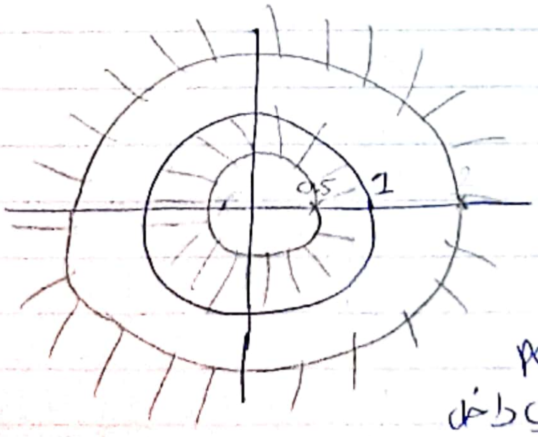
unit circule ←
لم يتخطى ال fourier transform موجود

* $X(Z)$ include unit circule $\Rightarrow$ FT exist

if $|Z| = 1$ $\Rightarrow$ unit step sequence.

e.g 3: $X(Z) = \frac{1}{(Z - 0.5)(Z - 2)}$

$\downarrow$ $|Z| > 0.5$

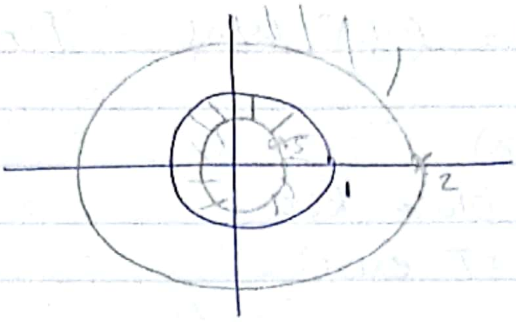


"Intersection"

ROC $\Rightarrow$ $|Z| > 2$.

(*) اذا كانت فيه ال poles
بلي تمثل ال ROC، في داخل
ال unit circle ، اذا بكون
انا مدرك انه تمثي ال
Fourier transform موجودة.

e.g.4: $X(z) = \dfrac{1}{(0.5-z)(z-2)}$



L16:-) Example: Consider the following signal

$x[n] = u[n]$, ① evaluate $X(z)$.

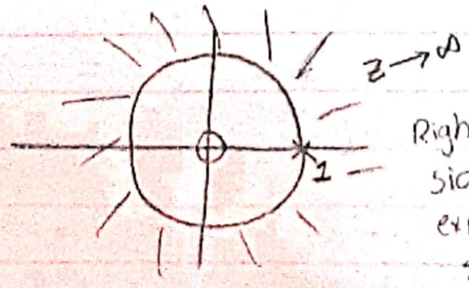
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} u[n]\, z^{-n}$$

$$= \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

② plot ROC.

$*\quad X(z) = \dfrac{(z-z_0)(z-z_1)\cdots}{(z-p_0)(z-p_1)\cdots}$ ← Zeros (o) ← poles (x)



Right sided exponential sequence

$z-1>0$

$|z|>1$

Example: the following signal

$x[n] = (\frac{1}{2})^n u[n]$ [right-sided exp. seq.]

1) evaluate $X(z)$
2) " and plot ROC
3) check if FT exist.

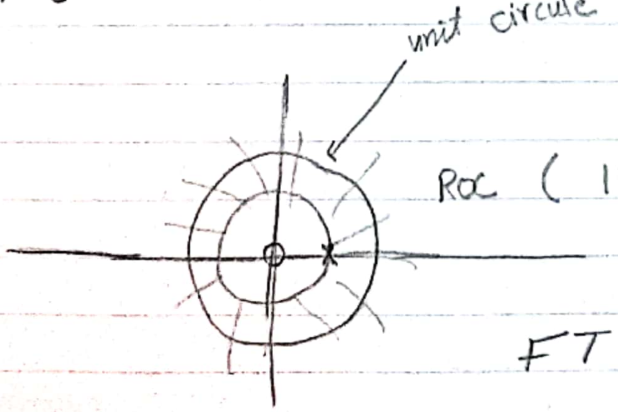
Ans:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} (\frac{1}{2})^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} (\frac{1}{2})^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (\frac{1}{2} z^{-1})^n = \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{z}{z-\frac{1}{2}}$$



ROC ($|z| > \frac{1}{2}$)

FT exist

Example 8=)) consider the following signal

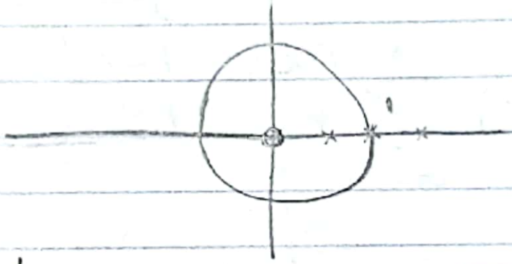
$$x[n] = a^n u[n]$$

① evaluate $x(z)$ ② plot ROC. ③ check if FT exist

$$x(z) = \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} (a z^{-1})^n$$

$$= \frac{1}{1 - a z^{-1}} = \frac{z}{z - a} \Rightarrow |z| > a$$



$\hookrightarrow$ $|a| < 1 \Rightarrow$ FT exist

$|a| = 1 \Rightarrow$ = FT 'exist'

$|a| > 1 \Rightarrow$ FT not exist.

[illegible] **Example:** Consider the following signal

$$x[n] = -a^n u[-n-1]$$

ⓐ evaluate $x(z)$ ⓑ determine and plot ROC

Ans:

$$x(z) = \sum_{n=-\infty}^{\infty} -a^n \underbrace{u[-n-1]} z^{-n} \longrightarrow u[-(n+1)]$$

$$= \sum_{n=-\infty}^{-1} -a^n z^{-n}$$

$$= \sum_{n=1}^{\infty} -a^{-n} z^n$$

$$= 1 - \sum_{n=0}^{\infty} a^{-n} z^{n}$$

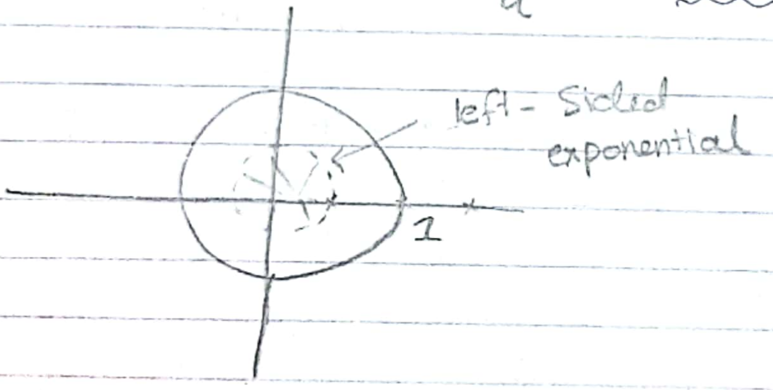
$$= 1 - \sum_{n=0}^{\infty} (a^{-1}z)^{n}$$

$$= 1 - \frac{1}{1-a^{-1}z}$$

$$= \frac{\cancel{1}-a^{-1}z-\cancel{1}}{1-a^{-1}z} = \underline{\frac{-a^{-1}z}{1-a^{-1}z}}$$

ROC: $1-a^{-1}z>0$

$1 > \frac{z}{a} \Rightarrow \boxed{a>z}$



↳ if $|a|<1 \Rightarrow$ FT not exist

Example: Considar a signal that the sum of two real exponential

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{-1}{3}\right)^n u[n]$$

a- evaluate $x(z)$

b- plot ROC

c- sepcify zeros and poles

Ans:

$$x(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\left(\frac{1}{2}\right)^n u[n] + \left(\frac{-1}{3}\right)^n u[n]\right] z^{-n}$$

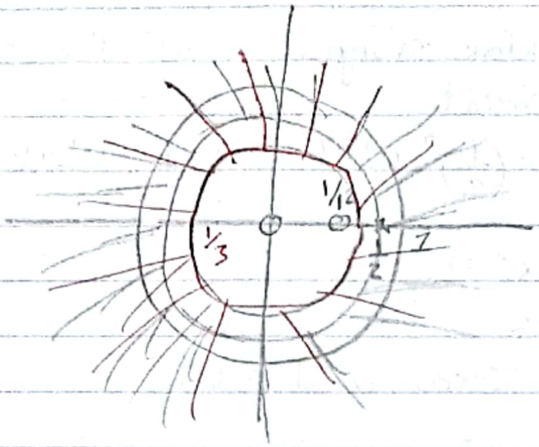
$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n]\, z^{-n} + \sum_{n=-\infty}^{\infty} \left(\frac{-1}{3}\right) u[n]\, z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n + \sum_{n=0}^{\infty} \left(\frac{-1}{3} z^{-1}\right)^n$$

$$= \frac{1}{1 - \frac{1}{2} z^{-1}} + \frac{1}{1 + \frac{1}{3} z^{-1}}$$

$$= \frac{1 + \frac{1}{3} z^{-1} + 1 - \frac{1}{2} z^{-1}}{\left(1 + \frac{1}{2} z^{-1}\right)\left(1 + \frac{1}{3} z^{-1}\right)}$$

$$= \frac{2z\left(z - 1/12\right)}{\left(z - 1/2\right)\left(z + 1/3\right)}$$

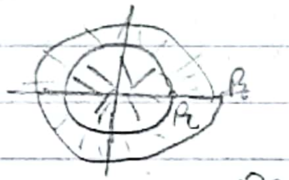


$Z \to \infty$ (Right sided)

ROC $\Rightarrow$ outer most.

* ___

* if $Z \to 0$



(left sided seq)

ROC $\Rightarrow$ inner most

imp

Example 3=1)

Consider the sequence

$$x[n] = \left(\frac{-1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[-n-1]$$

a- evaluate $x(z)$, b. plot ROC

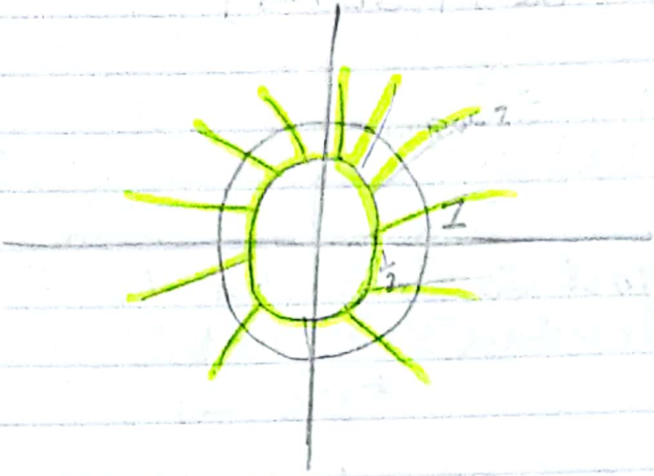
Ans 3=1)

$$x(Z) = \sum_{n=-\infty}^{\infty} x[n] \, Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\left(\frac{-1}{3}\right)^n u[n] \, Z^{-n} \right] + \sum_{n=-\infty}^{\infty} \left[\left(\frac{1}{2}\right)^n u[-(n+1)] \right]$$

$$= \sum_{n=0}^{\infty} \left(\frac{-1}{3} Z^{-1}\right)^n \quad \left[1 - \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{-n} Z^{n}. \right]$$

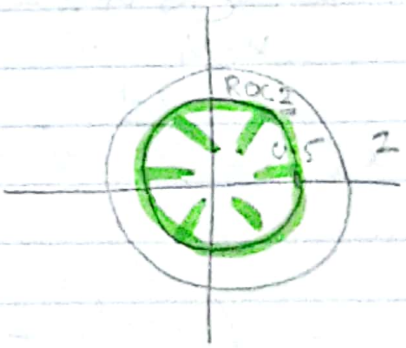
$$= \frac{1}{1+\frac{1}{3}z^{-1}} + 1 - \frac{1}{1-2Z}.$$

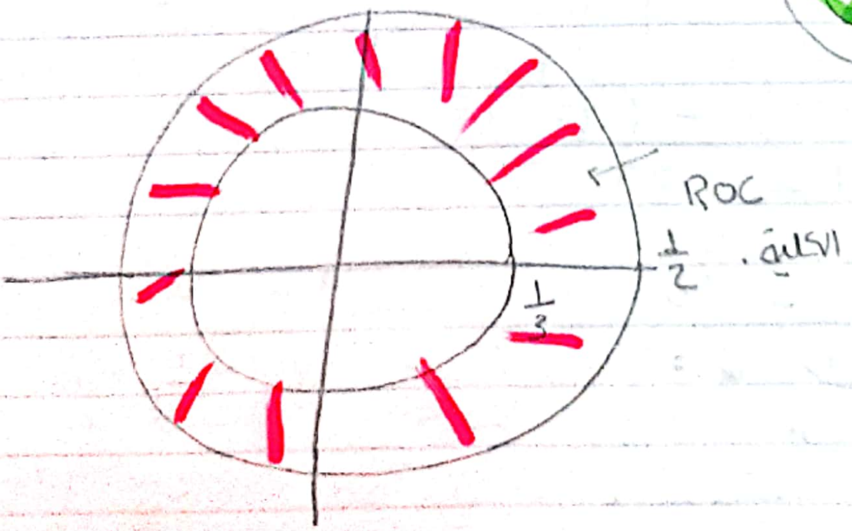


Zeros $= \frac{5}{12}$!

↳ pole #1
$- \; 1+\frac{1}{3}z^{-1} > 0 \Rightarrow |z| > \frac{-1}{3}$

↳ pole #2
$1-2Z > 0 \Rightarrow |z| < \frac{1}{2}.$





Finite sequence :=))

Example: $x[n] = \delta[n] + \delta[n-5]$

$$X[z] = 1 + z^{-5}$$
$$x[e^{j\omega}] = 1 + e^{-j5\omega}$$

If the signal is finite then the FT and the Z - transform both exist.

Example: $x[n] = \begin{cases} a^n , & 0 \leqslant n \leqslant N-1 \\ 0 , & o.w \end{cases}$ evaluate $x(z)$.

$$\star \quad \sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} \ominus \alpha^{N_2+1}}{1-\alpha} \quad ; N_2 \geqslant N_1$$

$$x(z) = \sum_{n=0}^{N-1} a^n z^{-n} = \sum_{n=0}^{N-1} (a z^{-1})^n$$

$$= \frac{(a z^{-1})^0 - (a z^{-1})^{N-1+1}}{1 - a z^{-1}}$$

$$= \frac{1 - (a z^{-1})^N}{1 - a z^{-1}} = \frac{1 - \frac{a^N}{z^n}}{1 - \frac{a}{z}}$$

$$= \frac{(z^n - a^n)}{z^n} \cdot \frac{z}{z-a}$$

$$= \frac{z^n - a^n}{z^{n-1}(z-a)} = \frac{z^n - a^n}{z^n - a z^{n-1}}$$

Example:=) another example if the previous example defind up to N.

$$x[n] = \begin{cases} a^n & , 0 \leqslant n \leqslant N \\ 0 & , o.w \end{cases}$$

$$X(z) = \sum_{n=0}^{N} a^N z^{-n} = \sum_{n=0}^{N} (a z^{-1})^n$$

$$= \frac{(a z^{-1})^0 - (a z^{-1})^{N+1}}{1 - a z^{-1}}$$

$$= \frac{(z^{n+1} - a^{n+1})}{z^{N+1}} \cdot \frac{z}{z-a}$$

$$= \frac{z^{n+1} - a^{n+1}}{z^{n+1} - z^n a}$$ No of Zeroes of N+1

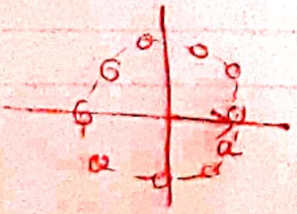
- The $\boxed{\text{finite signal}}$ has only poles at $\underline{z=0}$, or it may have No poles.
- but it has $\boxed{(N)\ \text{Zeros}}$

- The Region of convergence for a finite signal is the all Z-plane, in some case expt the origion, the infinity or both.

The Zeroes are $z^n = a^n$.

the Zeros complex are $= |z| e^{j(\frac{\overset{\phi}{2\pi}}{K})}$

$= a e^{j(\frac{2\pi}{K})}$, K = 0, 1, .. N-1



N = 8 as an example

Example: find the Z-transform and ROC of the sequence.

$$x[n] = \{ \overset{0}{\underset{\uparrow}{1}}, \overset{1}{0}, \overset{2}{3}, \overset{3}{-1}, \overset{4}{2} \}$$

Right-sided finite sequence.

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=0}^{4} x[n] z^{-n}$$

$$= x[0] + x[1] z^{-1} + x[2] z^{-2} + x[3] z^{-3} + x[4] z^{-4}$$

$$= 1 + 0 + 3z^{-2} + -z^{-3} + 2z^{-4}$$

ROC : all the values of <u>z except 0</u>

Example: $x[n] = \{ \overset{-4}{3}, \overset{-3}{-2}, \overset{-2}{-1}, \overset{-1}{0}, \overset{0}{\underset{\uparrow}{1}} \}$

Left-sided

$$X[z] = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-4}^{0} x[n] z^{-n} = z\{x[n]\}$$

$$= 3z^{+4} - 2z^{+3} - z^{+2} + 1$$

ROC :=)) all the values of <u>z except ∞</u>

$$* \sum_{k=0}^{n} r^{k} = \frac{r^{n+1} - 1}{r - 1}$$

example :=)

$$x[n] = \{\overset{-4}{2}, \overset{-3}{-1}, \overset{-2}{3}, \overset{-1}{2}, \underset{\uparrow}{\overset{0}{\underline{1}}}, \overset{1}{0}, \overset{2}{2}, \overset{3}{3}, \overset{4}{-1}\}$$

two sided

$$X(z) = \sum_{n=-4}^{4} x[n]\, z^{-n}$$

$$= 2z^{4} - z^{3} + 3z^{2} + 2z^{1} + 1 + 2z^{-2} + 3z^{-3} - z^{-4}$$

ROC: all the values of Z except <u>0 and ∞</u>

Causality and stability :=))

- Causality :=))

e.g : $y[n] - \frac{1}{2} y[n-1] = x[n]$

- $h[n] = 0 \quad , n < 0$

solutions ↓

$h[n] = a^n u[n]$ ⊛ Right sided sequence.

لو مازادت فيه n ، ح أضمن حافظ على ال bounded او عديه .

⊛ ROC is ~~outer~~ outside outer most pole.

- Stability:

* ROC includes ~~except~~ unit circuit

Example: Consider the following diffrence equation of the system

$$y[n] - \frac{1}{2} y[n-1] = x[n]$$

check the stability, and cousality of the system.

Ans:

$$y[n] - \frac{1}{2} y[n-1] = x[n]$$

Z transform: $Y(Z) - \frac{1}{2} Y(Z) Z^{-1} = X(Z)$

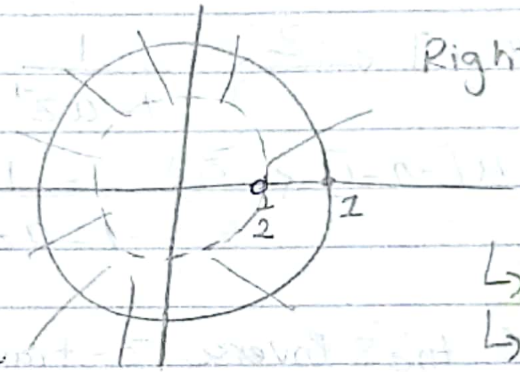
$$\left[1 - \frac{1}{2} Z^{-1}\right] Y(Z) = X(Z)$$

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{1}{1 - \frac{1}{2} Z^{-1}}$$

$\to |Z| > \frac{1}{2} \Rightarrow h[n] = a^n u[n]$

$\to |Z| < \frac{1}{2} , h[n] = -a^n u[-n-1]$

case 1:



Right sided

↳ causal

↳ stable

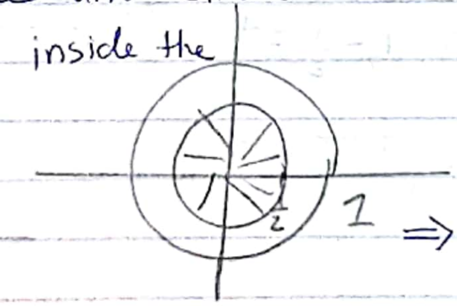
* ROC is to the outer ← the outer most pole.

⇒ system causal

*- ROC ~~include~~ the unit circle

+ All the poles inside the unit circle.

⇒ stable.



left side

⇒ ROC is to inner of the innermost pole

↳ non-causal

↳ non_causal

↳ unstable

⇒ Roc does not includes the unit circle

⇒ unstable

الملك
السجده
فصلت
الدخان
الواقعه
الحشر
الانسان

Inverse Z transform :=))

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

DFE

1. Inspection Method.

- $a^n u[n] \xleftrightarrow{Z} \frac{1}{1-az^{-1}}$; $\underline{|z|>|a|}$.
- $-a^n u[-n-1] \xleftrightarrow{Z} \frac{1}{1-az^{-1}}$; $\underline{|z|<|a|}$

Example:

evaluate the inverse z-transform of the

$$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}}, \boxed{|z| > 1/2}$$

$$\Rightarrow H[n] = \left(\frac{1}{2}\right)^n \underline{u[n]}$$ ✓

2. partial Fraction expansion :=))

M Zeros

$$X(z) = \frac{P(z^{-1})}{Q(z^{-1})} = \frac{\sum_{k=1}^{M} b_k z^{-k}}{\sum_{k=1}^{N} a_n z^{-k}} = \frac{z^N \sum_{k=1}^{M} b_k z^{M-k}}{z^M \sum_{k=1}^{N} a_k z^{N-k}}$$

N poles

- for $N > M \Rightarrow N-M$ Zeros at $z=0$
- for $N < M \Rightarrow M-N$ poles at $z=0$

$$X(Z) = \frac{b_0 \prod_{K=1}^{M} (1 - C_n Z^{-1})}{a_0 \prod_{K=1}^{N} (1 - d_k Z^{-1})}$$

Example: Consider asequence $x[n]$ with Z-transform

$$X(z) = \frac{1 \quad z^{0}}{(1 - \frac{1}{4} z^{-1})(1 - \frac{1}{2} z^{-1})}$$, evaluate $x[n]$.

$\boxed{|z| > 1/2}$

⇓

Right

← اذا كانت درجة
ال power لـ Z الموجودة
في البسط أقل من
درجة ال power الموجودة
في المقام لـ Z، أفضل
طريقة التعامل معها هي
partial fraction

← في حال كانت متساوية
أو أكبر منها، بنتعامل
بطريقة القسمة
الطويلة.

$$= \frac{A_1}{1 - \frac{1}{4} z^{-1}} + \frac{A_2}{1 - \frac{1}{2} z^{-1}}$$

$$1 = A_1 (1 - \frac{1}{2} z^{-1}) + A_2 (1 - \frac{1}{4} z^{-1})$$

when $z^{-1} = 2$

$$1 = A_1 (0) + A_2 (\tfrac{1}{2})$$

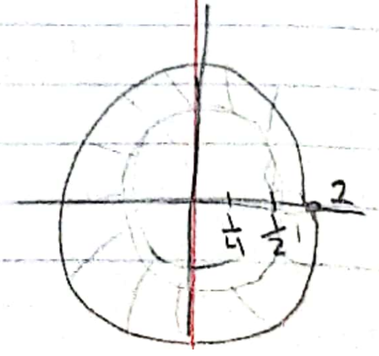
$$\Rightarrow \boxed{A_2 = 2}$$

when $z^{-1} = 4$

$$1 = A_1 (-1) + A_2(0) \Rightarrow \boxed{A_1 = -1}$$

$$\Rightarrow X(z) = \frac{-1}{1 - \frac{1}{4} z^{-1}} + \frac{2}{1 - \frac{1}{2} z^{-1}}$$

$$x[n] = -(\tfrac{1}{4})^{n} u[n] + 2(\tfrac{1}{2})^{n} u[n]$$



Example :-) Consider a sequence $x[n]$ with Z-transform

$$x(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$$

$$= \frac{(1+z^{-1})^{2}}{(1-\frac{1}{2}z^{-1})(1-z^{-1})} \quad ; \boxed{|z|>1} \Rightarrow$$

outer most [Right sided]

$$= \frac{A_1}{1-\frac{1}{2}z^{-1}} + \frac{A_2}{1-z^{-1}}$$

$$\Rightarrow \begin{array}{r|l} & 2 \\ \hline 1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2} & 1+2z^{-1}+z^{2} \\ & -2+3z^{-1}+-z^{2} \quad + \\ \hline & -1+5z^{-1} \end{array}$$

← بوقف لانه z^{-1} أقل من z^{-2}

$$x(z) = 2 + \frac{5z^{-1}-1}{(1-\frac{1}{2}z^{-1})(1-z^{-1})}$$

$$= 2 + \boxed{\text{Partial fract}}$$

$$\frac{5z^{-1}-1}{(1-\frac{1}{2}z^{-1})(1-z^{-1})} = \frac{A_1}{1-\frac{1}{2}z^{-1}} + \frac{A_2}{1-z^{-1}}$$

$$-1+5z^{-1} = A_1(1-z^{-1}) + A_2(1-\frac{1}{2}z^{-1})$$

when $z^{-1} = 2 \Rightarrow -1 + 10 = A_1(-1)$

$$\boxed{A_1 = -9}$$

when $Z^{-1} = 1$

$$-1+5 = A_2\left(\frac{1}{2}\right) \Rightarrow \boxed{A_2 = 8}$$

$$\Rightarrow X(Z) = 2 - \frac{9}{1-\frac{1}{2}Z^{-1}} + \frac{8}{1-Z^{-1}}$$

$$x(n) = 2\delta[n] - 9\left(\frac{1}{2}\right)^n u[n] + 8(1)^n u[n]$$

③ power Seriese Expansion:

مهم

$$X(Z) = \sum_{n=-\infty}^{\infty} x[n]\, Z^{-n}$$

Example: suppose $X(Z)$ is given ~~s~~ in the form

$$X(Z) = Z^2\left(1-\frac{1}{2}Z^{-1}\right)\left(1+Z^{-1}\right)\left(1-Z^{-1}\right)$$

evaluate $x[n]$

Ans: $X(Z) = Z^2\left(1-\frac{1}{2}Z^{-1}\right)\left(1+Z^{-1}\right)\left(1-Z^{-1}\right)$

$$= Z^2 - \frac{1}{2}Z^{-1} - 1 + \frac{1}{2}Z^{-1}$$

$$x_n = \begin{cases} 1, & n=-2 \\ -1/2, & n=-1 \\ -1, & n=0 \\ 1/2, & n=1 \\ 0, & \text{o.w} \end{cases}$$

$$\log(1+az^{-1}) = \sum_{n=1}^{\infty} \underbrace{\frac{(-1)^{n+1} a^{n} z^{-n}}{n}}_{x[n]}$$

$$\Rightarrow x[n] = 1\,\delta[n+2] + \underbrace{\tfrac{1}{2}\,\delta[n+1]}\, - \delta[n] \underbrace{- \tfrac{1}{2}\,\delta[n-1]}$$

Z transform properties :=))

1- linearity :=))

$$Z[a x_1[n] + b x_2[n]] = \sum_{n=-\infty}^{\infty} (a x_1[n] + b x_2[n])\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a x_1[n]\, z^{-n} + \sum_{n=-\infty}^{\infty} b\, x_2[n]\, z^{-n}$$

$$= \underset{\text{ROC }(R_x)}{a X_1(z)} + \underset{\text{ROC}(R_{x_2})}{b X_2(z)}$$

ROC = $R_x \cap R_{x_2}$

تقاطع

Example! Consider the following sequence

$$x[n] = \underbrace{\left(\tfrac{1}{2}\right)^{n} u[n]}_{x_1(n)} + \underbrace{\left(\tfrac{1}{3}\right)^{n} u[n]}_{x_2(n)}$$

evaluate $X[z]$.

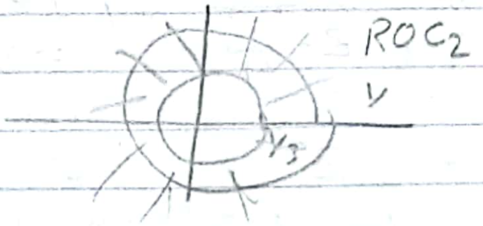
$$Z[x[n]] = Z[x_1[n] + x_2[n]]$$

$$= Z[x_1[n]] + Z[x_2[n]]$$

where: $Z[x_1[n]] = Z\left[\left(\tfrac{1}{2}\right)^{n} u[n]\right] = \dfrac{1}{1 - \frac{1}{2} z^{-1}}$, $|z| > \frac{1}{2}$

: ROC_1

$$Z[x_2[n]] = Z\left[\left(\tfrac{1}{3}\right)^n u[n]\right] = \frac{1}{1-\frac{1}{3}z^{-1}} \quad ; |z| > \frac{1}{3}$$



$$ROC = ROC_1 \cap ROC_2 = |z| > \frac{1}{2}$$

[Right- sided sequence]

② **time shifting** :

$$Z[x[n-n_0]] = \sum_{n=-\infty}^{\infty} x[n-n_0]\, z^{-n}$$

let $m = n - n_0 \Rightarrow n = m + n_0$

when $n = -\infty \Rightarrow m = -\infty$

when $n = \infty \Rightarrow m = \infty$

$$\Rightarrow Z[x[m]] = \sum_{m=-\infty}^{\infty} x[m]\, z^{-(m+n_0)} = z^{-n_0} \sum_{m=-\infty}^{\infty} x[m]\, z^{-m}$$

$$= \underline{\underline{X(z)\, z^{-n_0}}}$$

Example: Consider the Z-transform

$$X(z) = \frac{1}{z-\frac{1}{4}} \quad , \; |z| > \frac{1}{4}$$

$$= \frac{z^{-1}}{1-\frac{1}{4}z^{-1}} = \frac{1}{1-\frac{1}{4}z^{-1}} \cdot z^{-1}$$

$$x[n] = \left(\frac{1}{4}\right)^{n-1} u[n-1]$$

③ Multiplication by an Exponential sequence

$$z_0^n \, x[n]$$

$$Z\left[z_0^n x[n]\right] = \sum_{n=-\infty}^{\infty} z_0^n \, x[n] \, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \left(z/z_0\right)^{-n}$$

$$= X(z/z_0)$$

Example: Consider the following sequence

$$x[n] = r^n \cos(\omega_0 n) \, u[n]$$

evaluate $X(z)$.

Ans:

$$x[n] = r^n \cos(\omega_0 n) \, u[n]$$

$$= \frac{1}{2} r^n e^{j\omega_0 n} u[n] + \frac{1}{2} r^n e^{-j\omega_0 n} u[n]$$

$$= \frac{1}{2} \cdot \frac{1}{1 - re^{j\omega_0} z^{-1}} + \frac{1}{2} \cdot \frac{1}{1 - r e^{-j\omega} z^{-1}} \quad , \quad |z| > r$$

④ diffrentiation of $X(Z)$

$$n\,x[n] \xleftrightarrow{Z} -\boxed{Z}\,\frac{dX(z)}{dZ} \quad , \; ROC = R_x$$

proof :

$$X(Z) = \sum_{n=-\infty}^{\infty} x[n]\, Z^{-n}$$

$$\frac{d\,X(Z)}{dZ} = \sum_{n=-\infty}^{\infty} -n\,x[n]\, Z^{-n-1}$$

$$-Z\,\frac{dX(Z)}{dZ} = \sum_{n=-\infty}^{\infty} n\,x[n]\, Z^{-n}$$

$$= Z\,[n\,x[n]]$$

Example :=)) consider the following sequence

$$X(Z) = \log(1 + aZ^{-1}) \quad , \quad |Z| > |a|$$

Ans :=))

$$\frac{dX(z)}{dZ} = \frac{-aZ^{-2}}{1 + aZ^{-1}}$$

from differentiation property:

$$n\,x[n] \overset{Z}{\longleftrightarrow} -z\,\frac{dx(z)}{dz}$$

$$\Rightarrow \frac{-a z^{-2}}{1+az^{-1}} \cdot -z$$

$$= \frac{a z^{-1}}{1+a z^{-1}} \quad , \quad |z|>|a| \quad \Rightarrow x[n] = a \cdot a^{n-1} u[n-1]$$

(*) **Example:** using differentiation properity to determine the Z- transform of the sequence

$$x[n] = \underbrace{n\,a^n u[n]}$$

$$X(z) = -z \frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right) \quad , \quad |z|>|a|$$

$\left(\frac{1}{\text{س٥}-1}\right)$

$\frac{a\,+}{(\text{س٥}\ 1)}$

$$= -z\left(\frac{+az^{-2}}{(1-az^{-1})^2}\right) = \frac{az^{-1}}{(1-az^{-1})^2}$$

$$\Rightarrow n\,a^n u[n] \overset{Z}{\longleftrightarrow} \frac{+az^{-1}}{(1-az^{-1})^2} \quad , \quad |z|>|a|$$

⑤ Conjugation of a complex sequence:

(*) $$x^*[n] \overset{Z}{\longleftrightarrow} X^*(z^*) \qquad ROC = R_x$$

?

بتغير اشارة
ال
Immaginary
part.

⑥ Time Reversal 8=))

$$x^*[-n] \overset{Z}{\longleftrightarrow} x^*(1/z^*) \qquad ROC = \frac{1}{R_x}$$

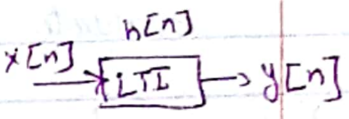
iR

Example:

⊛ using the time reversal property:

$$X(z) = \frac{1}{1-az} = \frac{-a^{-1}z^{-1}}{1-a^{-1}z^{-1}}, \quad |z| < |a^{-1}|$$

⑦ Convolution of sequence:



$y[n] = x[n] * h[n]$

$y(z) = x(z) \cdot H(z)$.

According to the Convolution property.

$$x_1[n] * x_2[n] \overset{Z}{\longleftrightarrow} X_1(z)\, X_2(z) \qquad ROC = R_{x_1} \cap R_{x_2}$$

To derive this property formally, we consider:

$$\underset{\sim\sim}{y[n]} = \sum_{k=-\infty}^{\infty} \underline{x_1[k]}\, \underbrace{x_2}_{}[n-k] \quad \overset{=h[n-k]}{}$$

$$Y(z) = \sum_{n=-\infty}^{\infty} y[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k] \right) z^{-n}$$

→ $m = n - k$
$n = m + k$
$k \to \infty \Rightarrow m \to \infty$
$k \to -\infty \Rightarrow m \to -\infty$

If we interchange the order of summation:

$$Y(z) = \sum_{k=-\infty}^{\infty} x_1[k] \sum_{n=-\infty}^{\infty} x_2[n-k]\, z^{-n}$$ ⊛

$n - k = m$

$$= \sum_{k=-\infty}^{\infty} x_1[k] \left\{ \sum_{m=-\infty}^{\infty} x_2[m] z^{-m} \right\} z^{-k}.$$

$$Y(z) = X_1(z)\, X_2(z)$$

Example 3:-

let $x_1[n] = a^n u[n]$ and $x_2[n] = u[n]$, evaluate the Z-transform of $y[n] = x_1[n] * x_2[n]$

$\boxed{\text{Ans 3:-}}$

$$y[n] = x_1[n] * x_2[n]$$

$$Y(z) = X_1(z) \cdot X_2(z)$$

$\hookrightarrow X_1(z) = \dfrac{1}{1 - az^{-1}}$, Assume $|z| > |a|$, $|a| < 1$

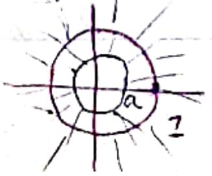
$\hookrightarrow X_2(z) = \dfrac{1}{1 - z^{-1}}$,

$$Y(z) = \frac{1}{1 - az^{-1}} \cdot \frac{1}{(1 - z^{-1})} = \frac{(+1)}{\left(\frac{1}{z} - \frac{a}{z}\right)\left(1 - \frac{1}{z}\right)}$$

$$= \frac{z^2}{(z-a)(z-1)}, \quad |z| > 1 \quad \Rightarrow Y(z) = \frac{1}{1-a}\left(\frac{1}{1-z^{-1}} - \frac{a}{1-az^{-1}}\right]$$

$$\hookrightarrow y[n] = \frac{1}{1-a}\left[u[n] - a\, a^n u[n]\right]$$

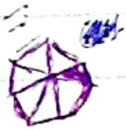
$$\Rightarrow \text{ROC} = |z| > 1$$



⑧ Initial value theorem:

if $x[n]$ is Zero for $n<0$, (i.e if $x[n]$ is causel) then

$$x[0] = \lim_{Z\to\infty} (X(Z))$$



Example: $x[n] = a^{-n} u[-n]$, evaluat $x(Z)$.

$$a^{n} u[n] \longleftrightarrow \frac{1}{1-aZ^{-1}}$$

$$= \frac{1}{1-a\left(\frac{1}{Z}\right)^{-1}} = \frac{1}{1-aZ}$$

Example: $X(Z) = \frac{1}{1-\frac{1}{2}Z^{-1}}$, find $x(0)$.

$$x[0] = \lim_{Z\to\infty} (X(Z))$$

Freq. Response: $H(e^{jw})$

& transfare function: $H(z)$

CHapter 5 :=))

Transfer analysis of LTI system.

$$x[n] \longrightarrow \boxed{\overset{h[n]}{LTI}} \longrightarrow y[n]$$

$y[n] = h[n] * x[n]$

$y(z) = h(z)\ x(z)$

$$Y(z) = X(z)\ \underbrace{H(z)}_{\text{transfer function}}$$

$$|Y(e^{jw})| = |X(e^{jw})|\ \underbrace{|H(e^{jw})|}_{\text{frequency Response}}$$

$$\measuredangle Y(e^{jw}) + \measuredangle X(e^{jw}) + \underbrace{\measuredangle H(e^{jw})}_{\text{negative غالبا}}$$

بتكون زاوية يعني . ← phase Vs delay

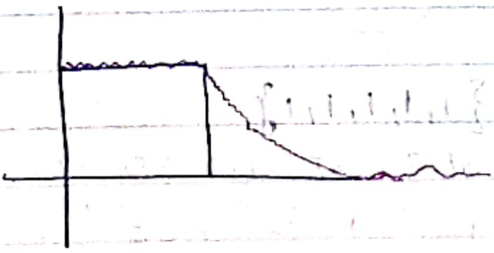
(degree) frequency ↵

↳ ~~time~~ "function" (time)

Example :=))

$$H_{LP} = \begin{cases} 1\ e^{-jwnd}, & |w| < w_c \\ 0 & w_c < |w| < \pi \end{cases}$$

$$h[n] = \frac{\sin(w_c(n-nd))}{\pi(n-nd)}.$$



$\measuredangle H_{LP}(e^{jw}) = -wnd$

linear phase.

$$H_{HP} = \begin{cases} 0, & |w| < w_c \\ 1, & w_c < |w| \leq \pi \end{cases}$$

$$\text{group delay} = \tau(w) = -\frac{d}{dw}\left(\measuredangle H(e^{jw})\right)$$

$$= nd.$$

↳ linear ⇒ group delay

↳ Constant

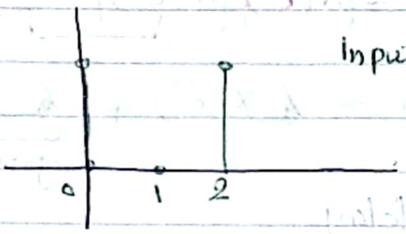
example: $h[n] = \delta[n-5]$. $\qquad \delta[n] \Rightarrow 1$

Ans :=))

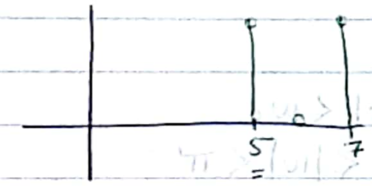
$$H(e^{jw}) = 1\, e^{-j5w}.$$

$$\measuredangle H(e^{jw}) = -5w$$

$$\tau(w) = 5 \text{ samples.}$$



$\Rightarrow$ output



"shift by 5"

$$\begin{array}{ccc} 0 & 1 & 2 \\ +5 & & \\ \hline 5 & 6 & 7 \end{array}$$

example :=))

$$h[n] = \frac{1}{5}\{1,1,1,1,1\}.$$

$$= 1/5\,[\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]]$$

$$H(e^{jw}) = \frac{1}{5}\left[e^{j0} + z^{-jw} + z^{-2jw} + z^{-3jw} + z^{-4jw}\right]$$

$$* \; e^{j\phi} = \cos(\phi) + j\sin(\phi).$$

$$= \frac{1}{5}\left(e^{2jw} + \underbrace{e^{jw} + e^{j0} + e^{-jw}}_{2\cos(w)} + e^{-2jw}\right) e^{-j2w}$$

$$* \; \cos(\phi) = \frac{e^{j\phi} + e^{-j\phi}}{2}$$

$$= \frac{1}{5}\underbrace{\left(1 + 2\cos(w) + 2\cos(2w)\right) e^{-j2w}}_{A\, e^{j\phi}}$$

$$\measuredangle H(e^{jw}) = -2w$$

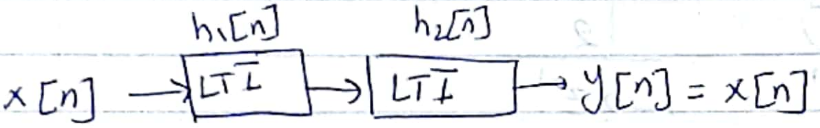
$$\tau(w) = 2 \text{ Samples}$$

* dB (decible :=))

↳ $dB = 20 \log(|H(e^{jw})|)$.

Inverse system 8=))

$h_2[n] = H_i[n]$



$h_1[n] * h_2[n] = \delta[n]$

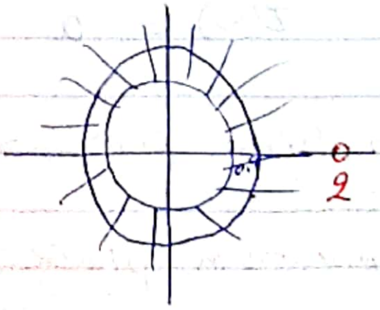
$H_1(Z) \cdot H_2(Z) = 1$

$\Rightarrow y[n] = x[n] = x[n] * \delta[n]$

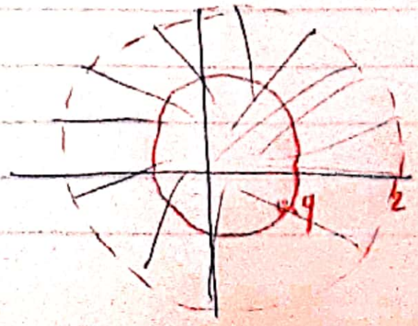
example 8=))

$$H(Z) = \frac{Z^{-1} - 0.5}{1 - 0.9Z^{-1}} \quad ; \quad |Z| > 0.9$$

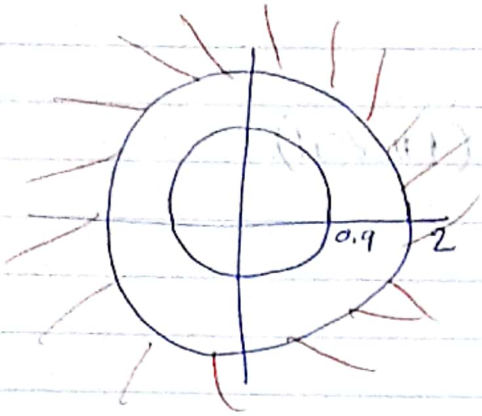
$$H_i(Z) = \frac{1 - 0.9Z^{-1}}{Z^{-1} - 0.5}$$



$|Z| < 2$.



left sidid.



$|z|>2$.

to find h[n] :-

$$H(z) = \frac{1.8 z^{-1}}{1-2z^{-1}} - \frac{2}{1-2z^{-1}}$$

$$\Rightarrow h[n] = -1.8\,(2)^{n-1}\,u[\overset{-(n-1)}{-n-1}] + 2\,(2)^{n}\,u[-n-1]$$

$$= -1.8\,(2)^{n-1}\,u[-(n-1)-1] + 2(2)^{n}\,u[-n-1]$$ left sided

$$\Rightarrow h[n] = 1.8\,(2)^{n-1}\,u[n-1] - 2\,(2^{n})\,u[n]$$ Right sided.

phase
1* linear phase system

$$\measuredangle H(e^{jw}) = -aw$$
$$\tau(w) = a$$

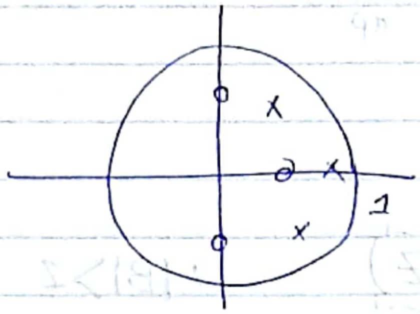
2- non-linear phase system

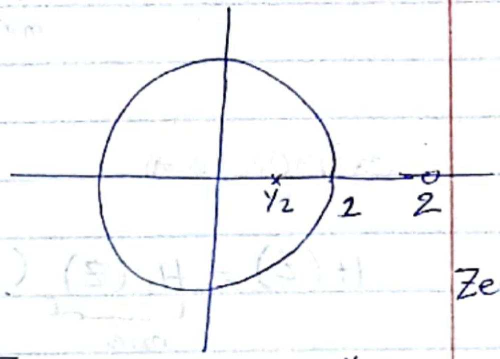
H(Z) Zero diagram
3- ϕ minimum phase system

4- all pass system.

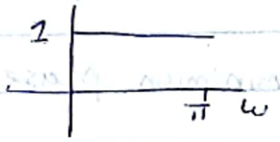
$\boxed{\text{Minimum phase} :\Rightarrow}$

$\boxed{\text{all } \overset{\text{pass}}{\cancel{\text{phase}}} \text{ system}}$





|Hap|.



Zeros' $\boxed{Z=2}$

pole: $2^* = 2$

$\frac{1}{2}$.

$$\boxed{H(Z) = H_{min} \cdot H_{all\,\overset{pass}{\cancel{phase}}}}$$

- Zero : $0.5\, e^{\sqrt{\pi/4}}$
- poles : ① $0.5\, e^{-j\pi/4}$

② $\frac{1}{0.5\, e^{-j\frac{\pi}{4}}} = 2\, e^{j\frac{\pi}{4}}$

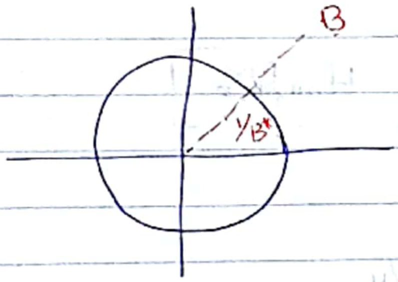
Minimum phase and All pass decomposition.

$$H(Z) = H_{min}(Z) \cdot H_{ap}(Z)$$

example

$$H(Z) = \underbrace{H_1(Z) \left(1 - \beta Z^{-1}\right)}_{min} \quad ; |B| > 1$$

; $H_1(Z)$: is the minimum phase system.



$$= H_1(Z) \cdot \left(1 - \beta Z^{-1}\right) \cdot \frac{1 - (1/\beta^*) Z^{-1}}{1 - \frac{1}{\beta^*} Z^{-1}}$$

$$H(Z) = \underbrace{H_1(Z) \cdot \left(1 - (1/\beta^*) Z^{-1}\right)}_{minimum} \underbrace{\frac{\left(1 - \beta Z^{-1}\right)}{\left(1 - \frac{1}{\beta^*} Z^{-1}\right)}}_{\text{all pass part.}}$$

$$H_{ap}(Z) = \frac{\left(Z^{-1} - C^*\right)}{\left(1 - C Z^{-1}\right)}$$

$$H(Z) = \underbrace{H_1(Z) \left(1 - \frac{1}{B^*} Z^{-1}\right) (-B)}_{Minimum.} \boxed{\frac{Z^{-1} - \frac{1}{B}}{1 - \frac{1}{B^*} Z^{-1}}} \quad \text{allpass}$$

Impulse Response for Rational system function :

DFE

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$\sum_{K=0}^{N} a_K Z^{-K} Y(Z) = \sum_{K=0}^{M} b_K Z^{-K} X(Z)$$

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{\sum_{K=0}^{M} b_K Z^{-K}}{\sum_{K=0}^{N} a_K Z^{-K}} \quad ; M \geq N$$

$$h(Z) = \sum_{r=0}^{M-N} B_r Z^{-r} + \sum_{K=1}^{N} \frac{A_K}{1-d_K Z^{-1}}$$

جزء ١ ↗ ← جزء ٢

~~$H(Z) = \sum_{r=0}^{M-N}$~~ $$h(n) = \sum_{r=0}^{M-N} B_r \delta(n-r) + \sum_{K=1}^{N} A_K (d_K)^n u[n]$$

types of filter

finite Impulse Response.

- H(Z) has no poles

e.g: $H(Z) = \sum_{K=0}^{M} b_n Z^{-K}$

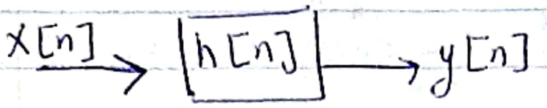
$$h[n] = \sum_{K=0}^{M} b_n \delta[n-k].$$

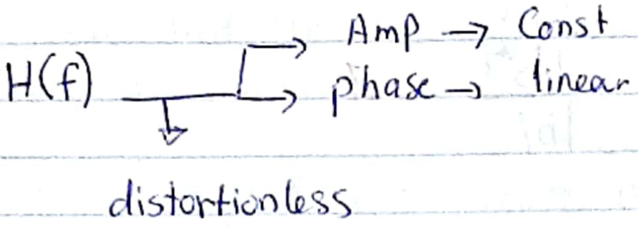
Infinite Impulse Response
(IIR)

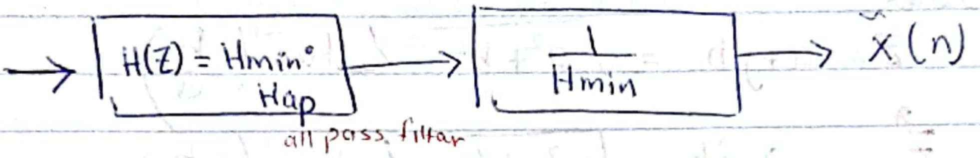
has at least one pole
is not cancel with Zero

$$H(Z) = \frac{1}{1-aZ^{-1}}$$

- feed back.







- All pass system $|H_{ap}(e^{j\omega})| = 1$

$$H_{ap}(Z) = \prod_{i=1}^{P} \frac{Z^{-1} - C_i^*}{1 - C_i Z^{-1}}$$

$C_i = r e^{j\phi}$

$C_i^* = r e^{-j\phi}$

$\frac{1}{C_i^*} = \frac{1}{r} e^{j\phi}$

↳ let $P = 1$

$$H_{ap}(Z) = \frac{Z^{-1} - C_i^*}{1 - C_i Z^{-1}}$$; since $Z = e^{j\omega}$

fourier transform

$$\Rightarrow H_{ap}(e^{j\omega}) = \frac{e^{-j\omega} - C_i^*}{1 - C_i e^{-j\omega}} = \frac{e^{-j\omega}(1 - C_i^* e^{j\omega})}{1 - C_i e^{-j\omega}}$$

$$\Rightarrow H_{ap}(e^{j\omega}) = \left| \frac{e^{-j\omega}(1 - C_i^* e^{j\omega})}{1 - C_i e^{-j\omega}} \right| = \frac{|e^{-j\omega}|\,|1 - C_i^* e^{j\omega}|}{|1 - C_i e^{-j\omega}|}$$

$$= \frac{1\,|1 - C_i^* e^{j\omega}|}{|1 - C_i e^{j\omega}|} \Rightarrow$$

given that

$|1 \cdot e^{-j\omega}| = 1$

if $b = 1 - c_i e^{-j\omega} \Rightarrow b^* = 1 - c_i e^{*j\omega}$

$$|H_{ap}(e^{j\omega})| = \frac{|b^*|}{|b|} = 1$$

ال mag. ثابت ← ال

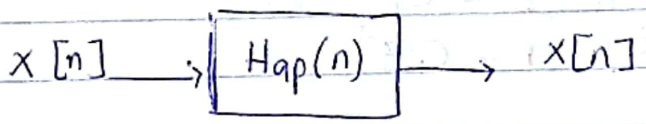
ال phase متغير

in general :

$$Z = a + jb = \sqrt{a^2 + b^2} \angle \tan^{-1}\left(\frac{b}{a}\right)$$

$$Z^* = a - jb = \sqrt{a^2 + b^2} \angle \tan^{-1}\left(\frac{-b}{a}\right)$$

زي ال buffer

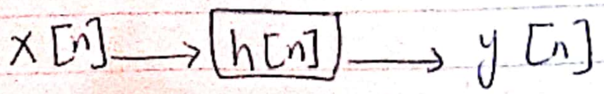


It can be noted that:

All pass system formed as product of:

$$\frac{Z^{-1} - C_i^*}{1 - C Z^{-1}}$$

- Minimum phase and All-Pass decomposition الفكرة



To factor $H(Z) = H_{min}(Z) \cdot H_{ap}(Z)$

1) Take Zero that lie the cutside $|Z| = 1$ (unit circuel) and move to Hap(Z).

2) Add poles to Hap(Z) in conjugate reccprocal location of Zeros

3) put Zeros Hmin(Z) to cancel poles added to Hap(Z)

examples=))
suppose $H(Z) = H_1(Z)(1 - BZ^{-1})$; $\underline{\underline{|B| > 1}}$, decompose $H(Z)$ into H_{min} and H_{ap}

Ans=))

$$H(Z) = H_1(Z)(1 - BZ^{-1})$$

$$= H_1(Z)\left(-B\left(Z^{-1} - \frac{1}{B}\right)\right) \cdot \frac{\left(1 - \frac{1}{B^*}Z^{-1}\right)}{\left(1 - \frac{1}{B^*}Z^{-1}\right)}$$

$$= \underbrace{H_1(Z)\left((-B)\left(1 - \frac{1}{B^*}Z^{-1}\right)\right)}_{H_{min}} \underbrace{\left(\frac{Z^{-1} - 1/B}{\cancel{Z}\,1 - \frac{1}{B^*}Z^{-1}}\right)}_{H_{ap}(Z)}$$

- Note:=)) the min-phase portion of any system has:
  - stable
  - causal
  - Inverse system.

$(1+j2)^* \Rightarrow (1-j2)$

example: decompose $H(Z)$ into min-phase and all pass

$$H(Z) = \left(1-\frac{1}{0.9}Z^{-1}\right)\left(1+\frac{1}{0.9\cancel{7}}Z^{-1}\right)\left(1-j0.7Z^{-1}\right)\left(1+j0.7Z^{-1}\right)$$

Ans: $\Rightarrow$

$$= -\left(\frac{1}{0.9}\right)\left(Z^{-1}-0.9\right)\left(\frac{1}{0.9}\right)\left(Z^{-1}+0.9\right)\left(1-j0.7Z^{-1}\right)\left(1+j0.7Z^{-1}\right)$$

- يوجد ال zero حلي بيكون أكبر من واحد

$$= -\left(\frac{1}{0.81}\right)\left(Z^{-1}-0.9\right)\left(\frac{1-0.9Z^{-1}}{1-0.9^*Z^{-1}}\right)\left(Z^{-1}+0.9\right)\left(\frac{1+0.9Z^{-1}}{1+0.9Z^{-1}}\right)\left(1-j0.7Z^{-1}\right)\left(1+j0.7Z^{-1}\right)$$

$$\Rightarrow H_{ap} = \frac{Z^{-1}-0.9}{1-0.9^*Z^{-1}} \cdot \frac{Z^{-1}+0.9}{1+0.9Z^{-1}}$$

$\frac{1}{0.9} > 1$

$$H_{min}(Z) = \frac{1}{0.81}\left(1-0.9Z^{-1}\right)\left(1+0.9Z^{-1}\right)\left(1-j0.7Z^{-1}\right)\left(1+j0.7Z^{-1}\right)$$

Example: Consider the sequence $H(Z)$ is given by

$$H(Z) = \frac{1+5Z^{-1}}{1+\frac{1}{2}Z^{-1}}$$, decompose $H(Z)$ into Hmin and Hap.

Ans: $\Rightarrow$

أكبر من صفر

$$H(Z) = \frac{1+5Z^{-1}}{1+\frac{1}{2}Z^{-1}} = \frac{\overset{\uparrow}{5}\left(Z^{-1}+1/5\right)}{1+\frac{1}{2}Z^{-1}} \cdot \frac{1+\left(\frac{1}{5}\right)^*Z^{-1}}{1+\left(\frac{1}{5}\right)^*Z^{-1}}$$

$$= \underbrace{5 \cdot \frac{1}{1+\frac{1}{2}Z^{-1}} \cdot 1+\frac{1}{5}Z^{-1}}_{H_{min}} \cdot \underbrace{\frac{Z^{-1}+1/5}{1+\frac{1}{5}Z^{-1}}}_{H_{ap}}$$

$(-2j)(5j)$

$(-10)(-1)$

$\underline{10}$

CH6: Structure for discrete time system.

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$H(z) = \frac{\sum_{i=0}^{M} b_k z^{-k}}{\sum_{i=0}^{N} a_k z^{-k}}$$

Simulink model:

① $x[n] \longrightarrow \boxed{z^{-1}} \longrightarrow x[n-1].$

$x(z) z^{-1}.$ $\Longrightarrow$ $y(n) = x(n) * h(n)$

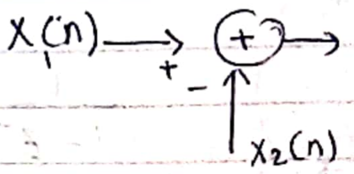
$y(z) = x(z)\ H(z).$

$y(z) = z^{-1} x(z);$

$\hookrightarrow y[n] = x(n-1)$

$$x[n] \text{ --- } \boxed{z^{-1}} \text{ --- } \boxed{z^{-1}} \text{ --- } x(n-2)$$

②



$y(n) = x_1(n) + x_2(n)$

③ $x(n) \text{ --- } \boxed{b_0} \text{ --- } b_0 x(n)$ $\equiv$ $\underrightarrow{x(n)\quad b_0}$ $\Rightarrow x(n).b_0$

Let $y[n] = \sum_{k=0}^{N} x[k] = x[n] + \sum_{k=0}^{N-1} x[k]$; $y[n-1] = \sum_{k=0}^{n-1} x[k]$

$$y[n] = x[n] + \sum_{k=1}^{n} x[k]$$

$$y[n] = x[n] + y[n-1]$$

$$\Rightarrow y[n] - y[n-1] = x[n]$$

In general:-

$$y[z] - \sum_{k=1}^{N} a_k z^{-1} Y(z) = \sum_{k=0}^{M} b_k z^{-1} X(z).$$

$$Y(z) = \sum_{k=0}^{M} b_k z^{-1} x(z) + \sum_{k=1}^{N} a_k z^{-1} Y(z).$$

$$\Rightarrow \left[1 - \sum_{k=1}^{N} a_k z^{-1}\right] Y(z)$$

$$= \sum_{k=0}^{M} b_k z^{-k} x(z).$$

$$\Rightarrow \boxed{\frac{Y(z)}{x(z)} = H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}}$$

$$\frac{1}{1 - z^{-1}}$$

example (CH5) :=))

$5>1$

$$H(z) = \frac{1+j5z^{-1}}{1+\frac{1}{2}z^{-1}} \Rightarrow \frac{j5\left(z^{-1}+1/j5\right)}{1+\frac{1}{2}z^{-1}} \cdot \frac{1+\left(\frac{1}{j5}\right)^{*}z^{-1}}{1+\left(\frac{1}{j5}\right)^{*}z^{-1}}$$

$$= \frac{j5\left(z^{-1}-j/5\right)}{1+\left(\frac{1}{2}\right)z^{-1}} \cdot \frac{1+\left(-j/5\right)^{*}z^{-1}}{1+\left(-j/5\right)^{*}z^{-1}}$$

$$= \frac{j5\left(z^{-1}-j/5\right)}{1+\left(\frac{1}{2}\right)z^{-1}} \cdot \frac{1+\left(j/5\right)z^{-1}}{1+\left(j/5\right)z^{-1}}$$

H_{min}

$H_{ap}(z)$

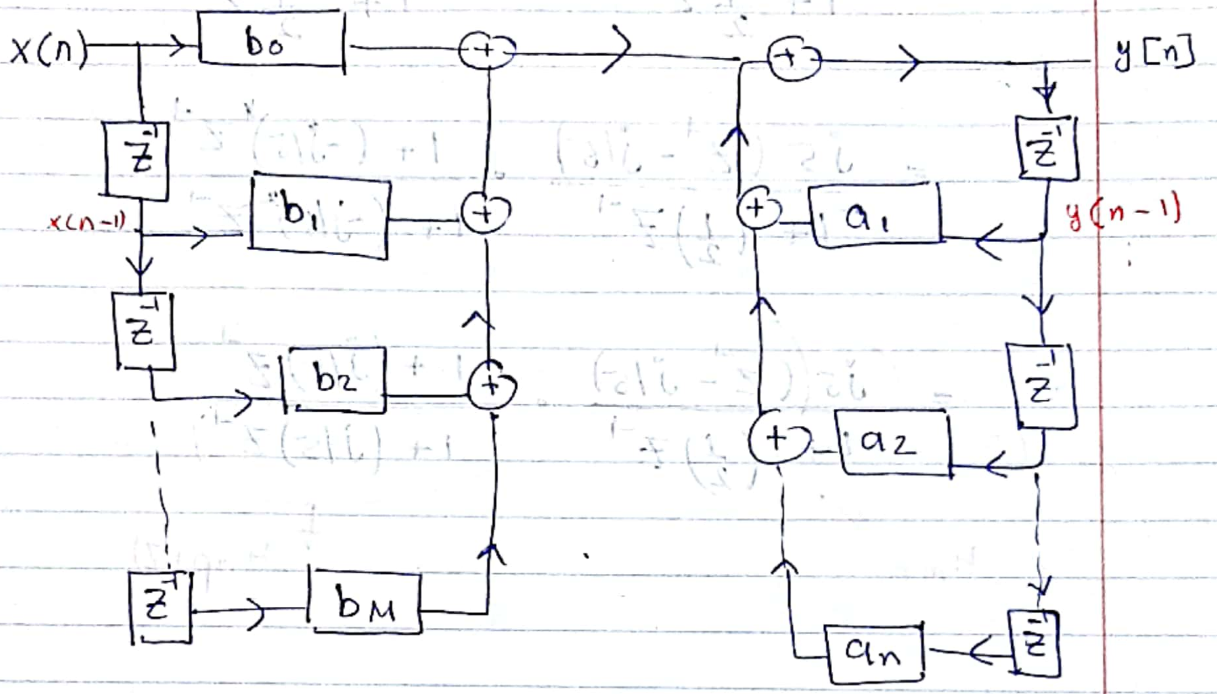
CH6:

$$\Rightarrow \quad Y(z)\left[1-\sum_{k=1}^{N} a_k z^{-k}\right] = X(z)\sum_{k=0}^{M} b_k z^{-k}.$$

$$\Rightarrow \quad Y(z) = \sum_{k=0}^{M} b_n z^{-k}\, X(z) + \sum_{k=1}^{N} a_k z^{-k}\, Y(z)$$

Inverse $\Rightarrow$ $$y(n) = \sum_{k=0}^{M} b_k\, x(n-k) + \sum_{k=1}^{N} a_k\, y(n-k)$$

$$y(n) = \sum_{K=0}^{M} b_K \, x(n-k) + \sum_{K=1}^{N} a_K \, y(n-k) .$$



(Direct Form I)

$$H(Z) = \frac{\sum_{K=0}^{M} b_K Z^{-K}}{1 - \sum_{K=1}^{N} a_K Z^{-K}} = \left(\frac{1}{1 - \sum_{K=1}^{N} a_K Z^{-K}}\right)\left(\sum_{K=0}^{M} b_K Z^{-K}\right)$$

$$\frac{Y(Z)}{X(Z)} = H(Z) = H_1(Z) \cdot H_2(Z)$$

$\hookrightarrow$ $H_1(Z) = \dfrac{1}{1 - \sum_{K=1}^{N} a_K Z^{-K}}$ and $H_2(Z) = \sum_{K=0}^{M} b_K Z^{-K}$

$$\Rightarrow \quad W(Z) = H_1(Z) \cdot X(Z).$$

$$Y(Z) = H_2(Z) \cdot W(Z) \; ; \quad \frac{Y(Z)}{X(Z)} = H_1(Z) . H_2(Z)$$

$$= \underbrace{H_1(Z) . X(Z)}_{W(Z).} . H_2(Z)$$

⇒ Since:

$$H_1(Z) = \frac{1}{1 - \sum_{K=1}^{N} a_K Z^{-K}} \; ; \quad W(Z) = \frac{1}{1 - \sum_{K=1}^{N} a_K Z^{-K}} X(Z)$$

$$\Rightarrow \quad X(Z) = \left[1 - \sum_{K=1}^{N} a_K Z^{-k}\right] W(Z)$$

$$W(Z) = X(Z) + \sum_{K=1}^{N} a_K Z^{-K} W(Z)$$

$$w(n) = w(n) + \sum_{K=1}^{N} a_K w(n-K). \quad \text{——— (1)}$$

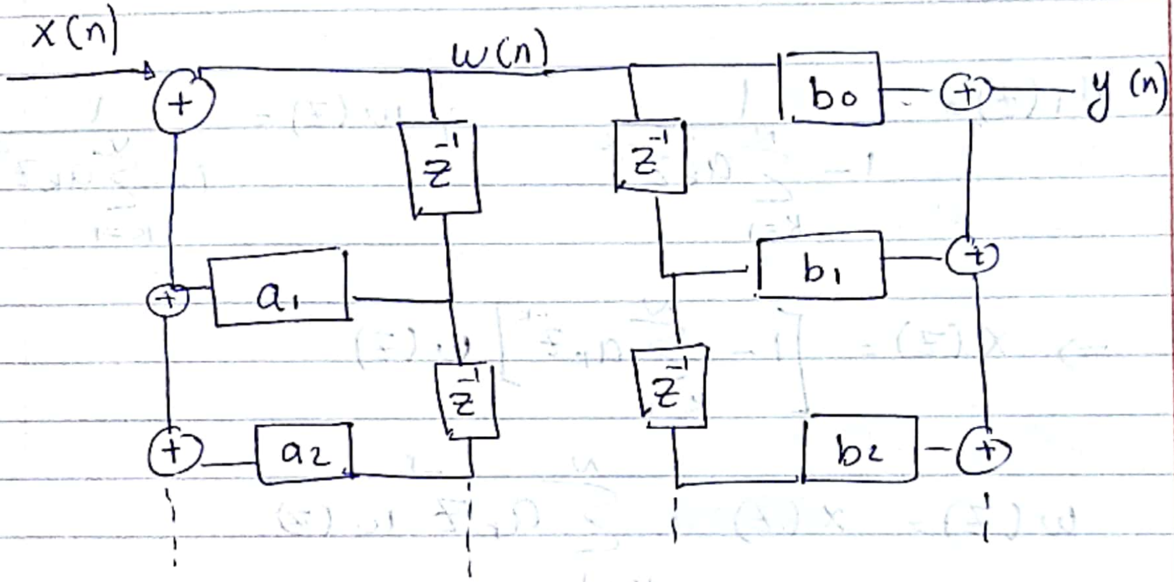
$$Y(Z) = H_2(Z) \; W(Z)$$

$$= \sum_{K=0}^{M} b_K Z^{-K} \; W(Z)$$

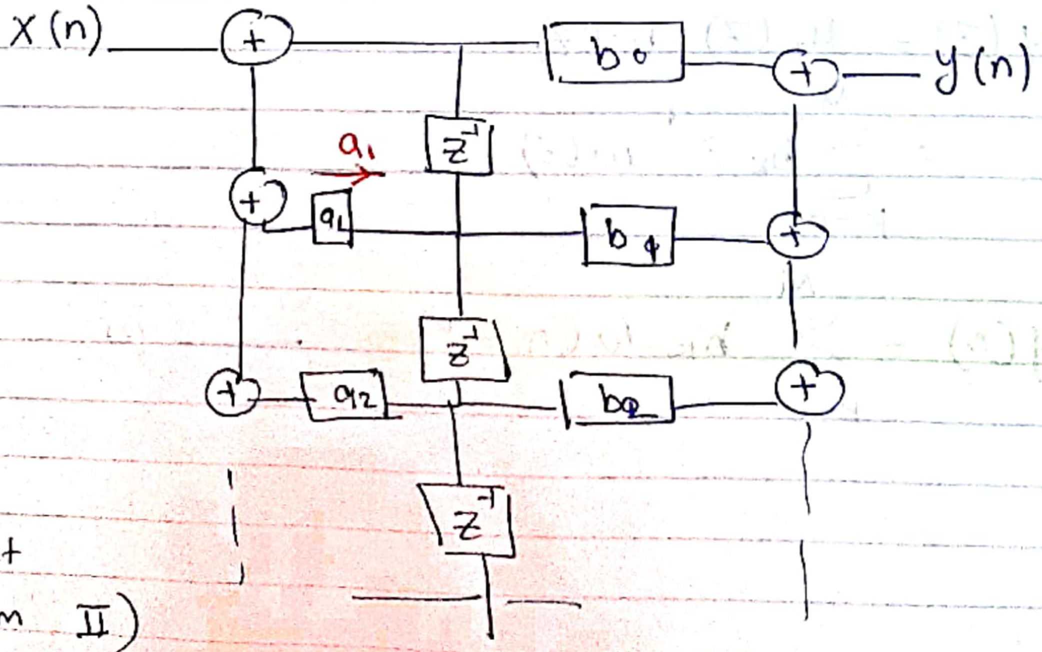
$$y(n) = \sum_{K=0}^{M} b_K w(n-k) \quad \text{——— (2)}$$

$$\Rightarrow \quad w(n) = x(n) + \sum_{k=1}^{N} a_k \, w(n-k)$$

$$y(n) = \sum_{k=0}^{M} b_k \, w(n-k)$$



$\Rightarrow$



(direct form II)

canonical form.

Example 8=11 Consider the following transfer function of the system:

$$H(Z) = \frac{1+2Z^{-1}}{1-1.5Z^{-1}+0.9Z^{-2}} \rightarrow$$ طالما في عندي Poles ⇐ (IIR)

Draw: 1) direct form I
2) direct form II.

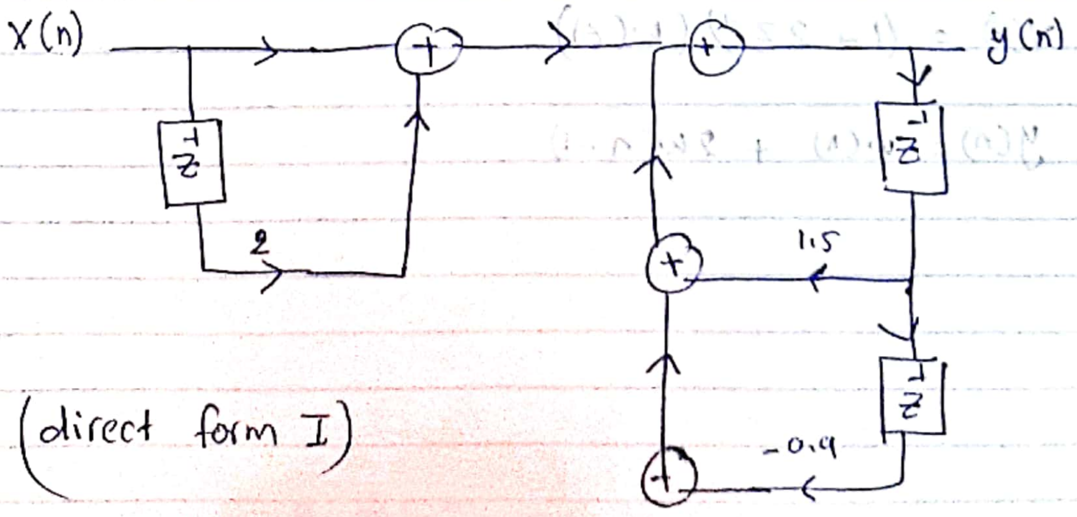
to draw direct form I 8=11

$$\frac{Y(Z)}{X(Z)} = \frac{1+2Z^{-1}}{1-1.5Z^{-1}+0.9Z^{-2}}$$

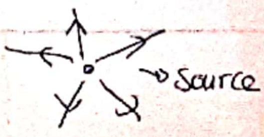
$$Y(Z)\left[1-1.5Z^{-1}+0.9Z^{-2}\right] = \left[1+2Z^{-1}\right]X(Z)$$

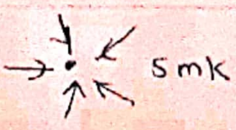
$$Y(Z) = \left(1+2Z^{-1}\right)X(Z) + \left(1.5Z^{-1}-0.9Z^{-2}\right)Y(Z)$$

$$y(n) = x(n) + 2x(n-1) + 1.5y(n-1) - 0.9y(n-2).$$



(direct form I)





direct form II :=))

$$H(Z) = \frac{1+2Z^{-1}}{1-1.5Z^{-1}+0.9Z^{-2}} =$$

$$= \left(\frac{1}{1-1.5Z^{-1}+0.9Z^{-2}}\right)\left(1+2Z^{-1}\right)$$

$$= H_1(Z) \cdot H_2(Z)$$

$$w(Z) = H_1(Z) \cdot X(Z)$$

$$= \left(\frac{1}{1-1.5Z^{-1}+0.9Z^{-2}}\right) X(Z)$$

$$w(Z)\left[1-1.5Z^{-1}+0.9Z^{-2}\right] = X(Z)$$

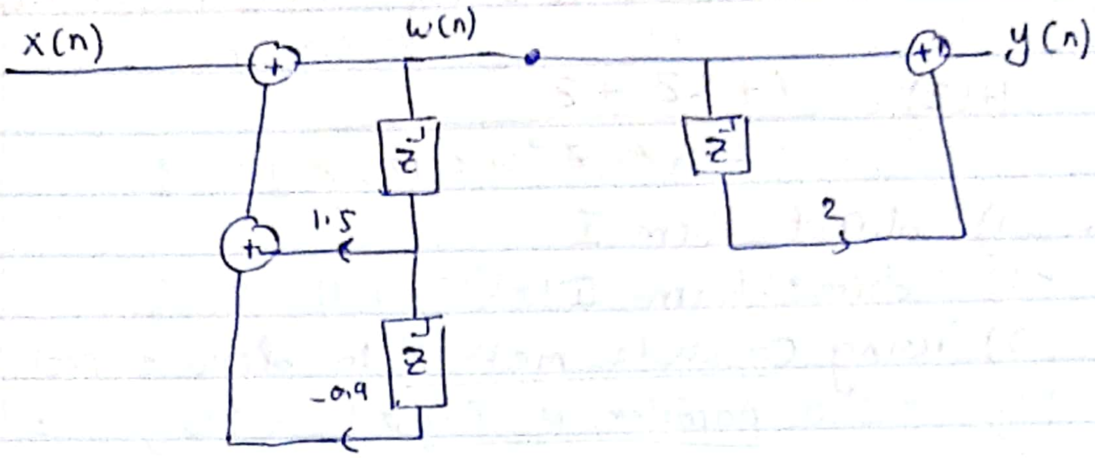
$$w(n) - 1.5\,w(n-1) + 0.9w(n-2) = x(n)$$ ✓

$$w(n) = x(n) + 1.5\,w(n-1) - 0.9w(n-2)$$

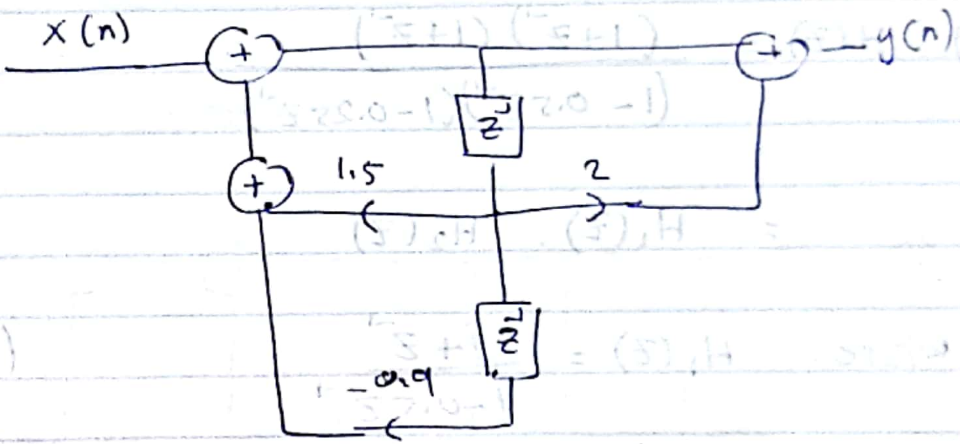
$$Y(Z) = H_2(Z)\,w(Z)$$

$$Y(Z) = (1+2Z^{-1})(w(Z))$$

$$y(n) = w(n) + 2w(n-1)$$



لازم
أبسطها
عشان
اوحد
direct
form II



direct form II

Example: Consider the following transfer function

$$H(Z) = \frac{1+2Z^{-1}+Z^{-2}}{1-0,75Z^{-1}+0,125Z^{-2}}$$

Draw: 1) direct form I
✓2) direct form II
3) using cascade method to draw direct I
4) 〃 paraller 〃 〃 〃 〃 〃

Ans:=)

3) $$H(Z) = \frac{(1+Z^{-1})(1+Z^{-1})}{(1-0.5Z^{-1})(1-0.25Z^{-1})}$$

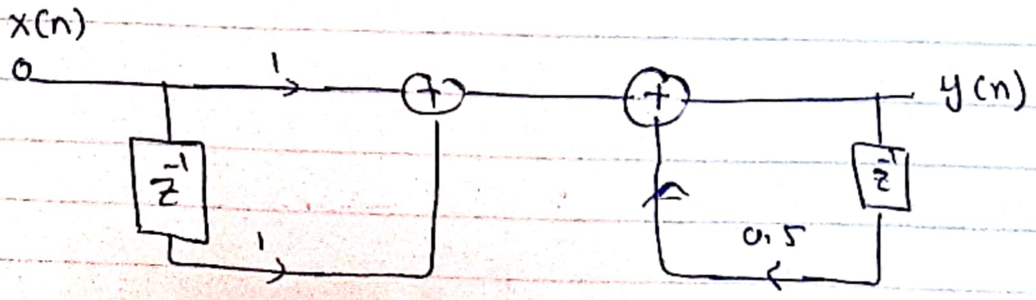
$$= H_1(Z) \cdot H_2(Z)$$

where: $H_1(Z) = \frac{1+Z^{-1}}{1-0.5Z^{-1}}$ (direct form I)

$$\frac{Y(Z)}{X(Z)} = \frac{1+Z^{-1}}{1-0,5Z^{-1}} \Rightarrow Y(Z)\left[1-0,5Z^{-1}\right] = X(Z)\left[1+Z^{-1}\right]$$

$$\Rightarrow Y(Z) = X(Z)\left[1+Z^{-1}\right] + 0,5Z^{-1}Y(Z)$$

$$y(n) = x(n) + x(n-1) + 0,5\,y(n-1).$$



$$H_2(Z) = \frac{1+Z^{-1}}{1-0.25Z^{-1}} = \frac{y(Z)}{X(Z)}$$

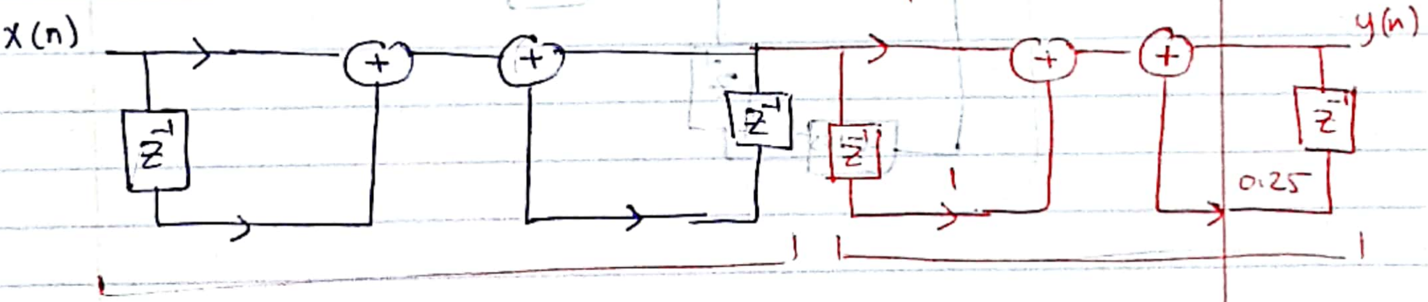
$$y(Z)\left[1-0.25Z^{-1}\right] = X(Z)\left[1+Z^{-1}\right]$$

$$y(Z) = (1+Z^{-1})X(Z) + 0.25Z^{-1}y(Z)$$

$$y(n) = y_1(n) + y_1(n-1) + 0.25y(n-1)$$

(final form)

(cascade)



④ parallel method 8=))

$$H(Z) = \frac{1+2Z^{-1}+Z^{-2}}{1-0.75Z^{-1}+0.125Z^{-2}}$$

دا قسمة طويلة $= 8 + \frac{-7+8Z^{-1}}{1-0.75Z^{-1}+0.125Z^{-2}}$

↳ By using direct form I :=))

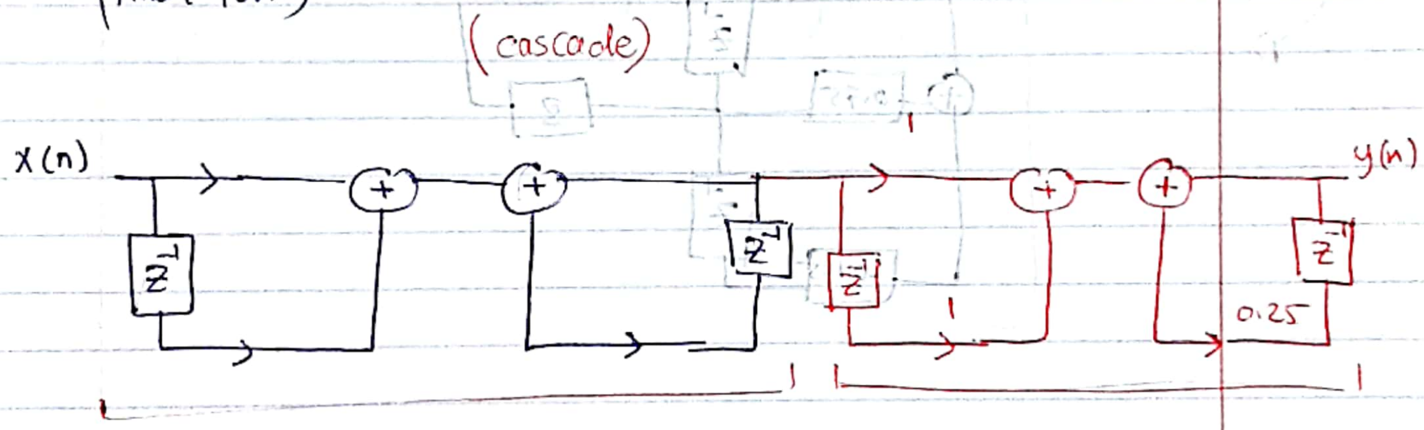
$$H_2(z) = \frac{1+z^{-1}}{1-0.25z^{-1}} = \frac{y(z)}{x(z)}$$

$$y(z)\left[1-0.25z^{-1}\right] = x(z)\left[1+z^{-1}\right]$$

$$y(z) = (1+z^{-1})x(z) + 0.25z^{-1}y(z)$$

$$y(n) = y_1(n) + y_1(n-1) + 0.25\,y(n-1)$$

(final form)



④ parallel method 8⇒))

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$$

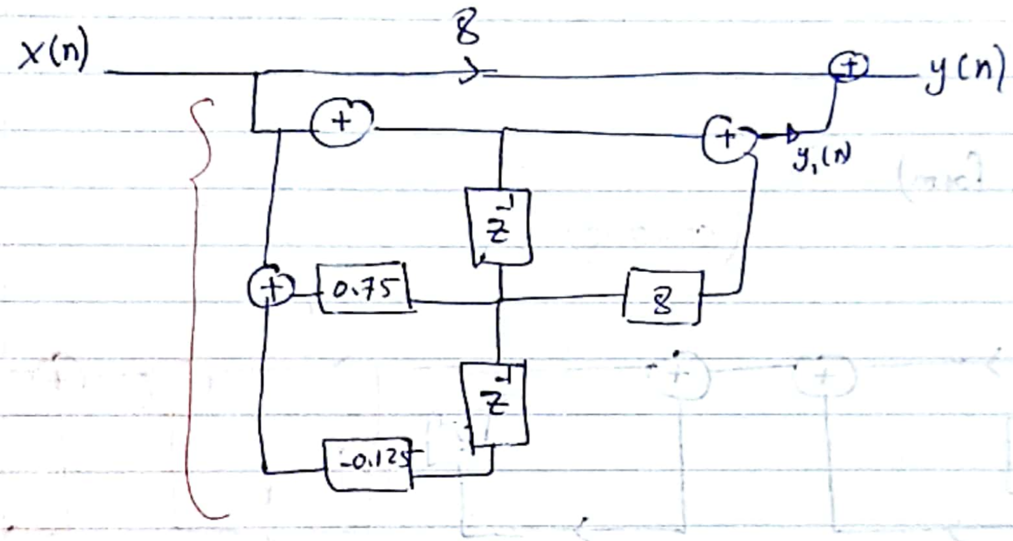
↳ قسمة طويلة $= 8 + \dfrac{-7+8z^{-1}}{1-0.75z^{-1}+0.125z^{-2}}$

↳ By using direct form I :⇒1

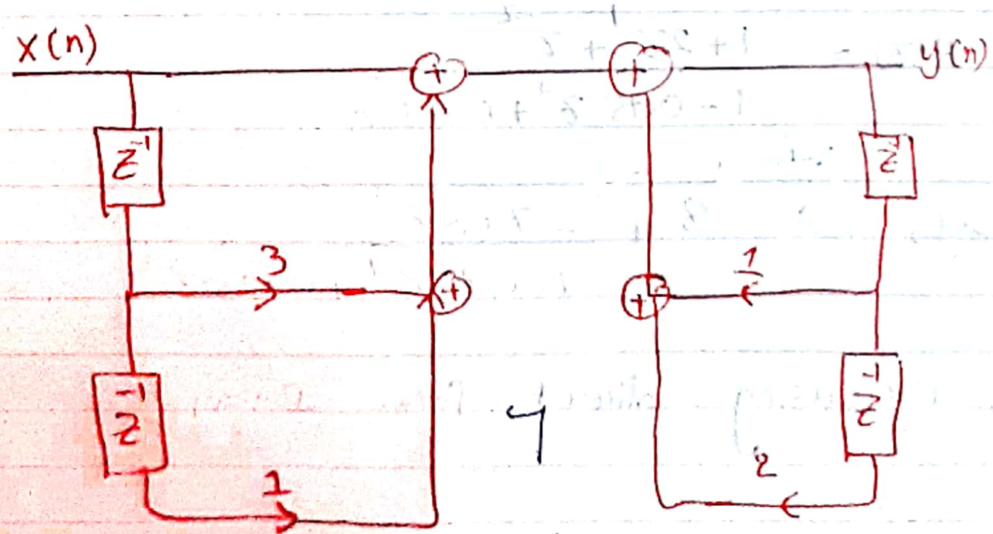
$$\frac{Y(z)}{X(z)} = 8 + \frac{-7+8z^{-1}}{1-0.75z^{-1}+0.125z^{-2}}$$

$$Y(z) = 8X(z) + \underbrace{\frac{-7+8z^{-1}}{1-0.75z^{-1}+0.125z^{-2}} \cdot X(z)}_{y_1(n)}$$

direct form II :=))



Example :=)) Consider the following system



بما انه في

طرفي المنطقة في sum (direct form I) ؛

ⓐ specify the type of structure form.

direct form I,

ⓑ evaluate the DFE of the system.

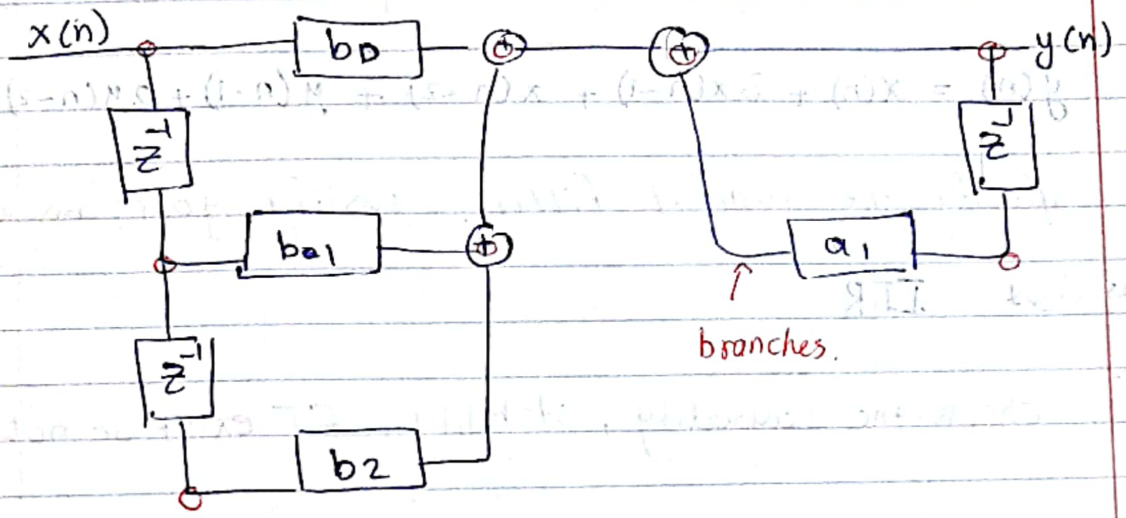
$$y(n) = x(n) + 3x(n-1) + x(n-2) + y(n-1) + 2y(n-2)$$

ⓒ specify the type of filter. justify your answer.

لانو في poles   IIR

ⓓ check the causality, stability, FT exist or not.

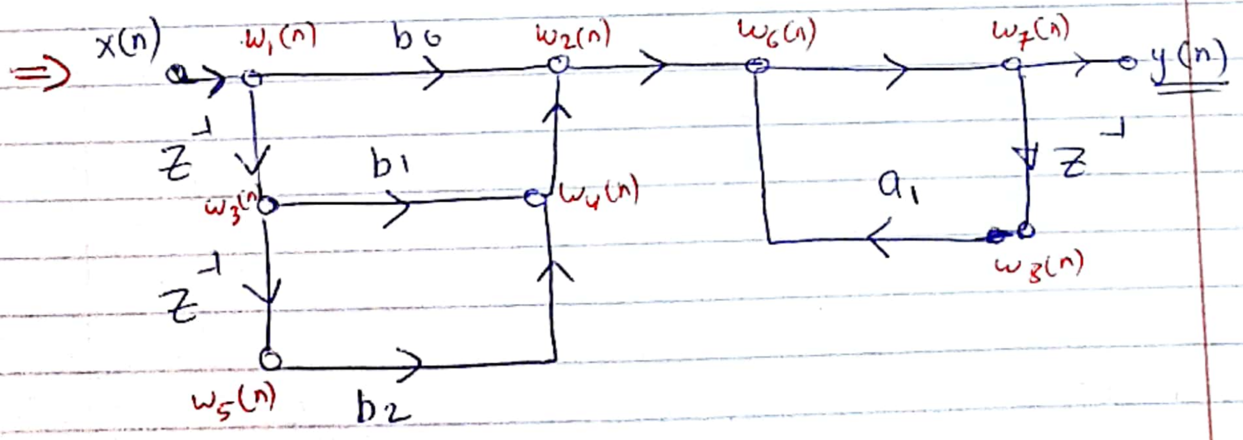
Signal flow graph



(nodes) مكان التفرعات اه

$\boxed{y(n) = w_7(n)}$

⇒



الداخل عليه

$w_1(n) = x(n)$

$w_2(n) = b_0 w_1(n) + w_4(n)$

$w_3(n) = w_1(n-1)$

$w_4(n) = b_1 w_3(n) + b_2 w_5(n)$

$w_5(n) = w_3(n-1)$

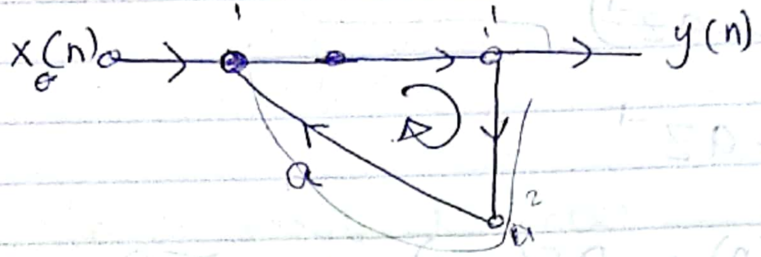
$w_6(n) = w_2(n) + a_1 w_8(n)$

$w_7(n) = w_6(n)$

$w_8(n) = w_7(n-1)$

CH6 - Part 2

feedback in IIR system :=))

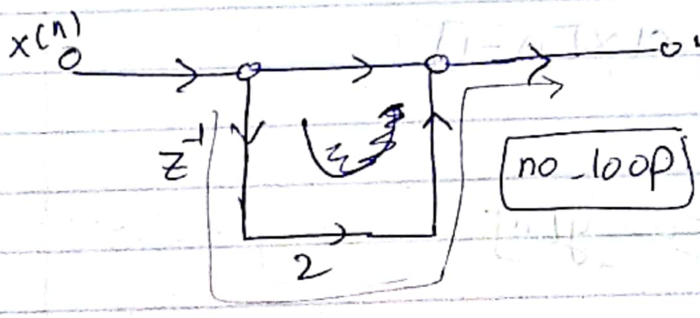


$\longrightarrow 1, a, a^2, \dots a^n$

output.

$$y(n) = a y(n-1) + x(n)$$

e.g: $y(n) = x(n) - 2x(n-1)$



$h[n] = a^n u[n] \Rightarrow$ IIR

$$h[n] = \begin{cases} a^n, & 0 \leq n \leq N-1 \\ 0, & o.w \end{cases}$$ FIR

e.g: $H(z) = \frac{1 - a^2 z^{-2}}{1 - az^{-1}}$

Ans:

$$= \frac{(\cancel{1-az^{-1}})(1+az^{-1})}{(\cancel{1-az^{-1}})} = 1+az^{-1}$$

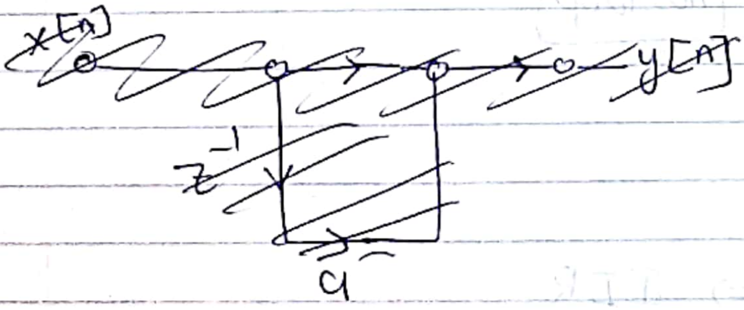
$$H(z) = 1 + az^{-1}$$

$$h[n] = \delta(n) + a\delta(n-1).$$ → FIR system

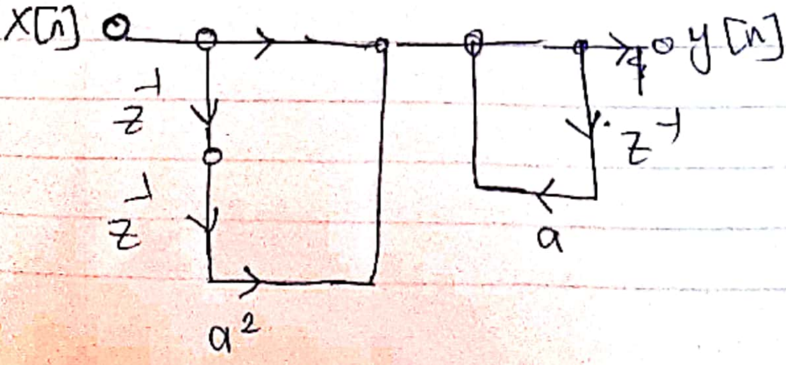
$$\frac{y(z)}{x(z)} = \frac{1+az^{-1}}{1}$$

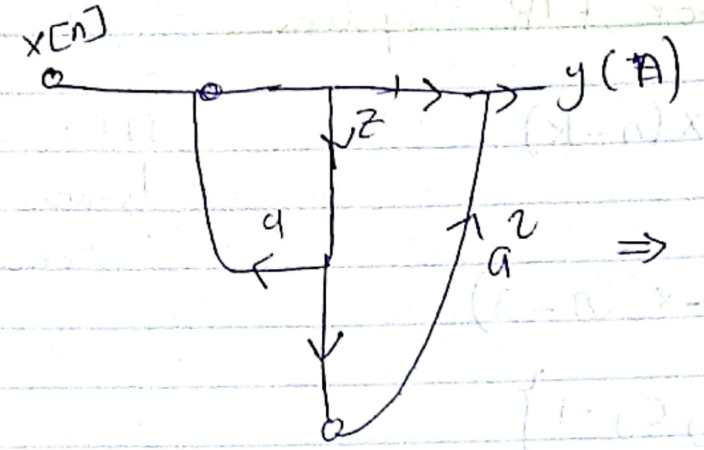
$$y(z) = x(z) + ax(z)z^{-1}$$

$$y[n] = x[n] + ax[n-1]$$



$$y[n] = ay[n-1] \cancel{=} + x[n] - a^2 x[n-2]$$

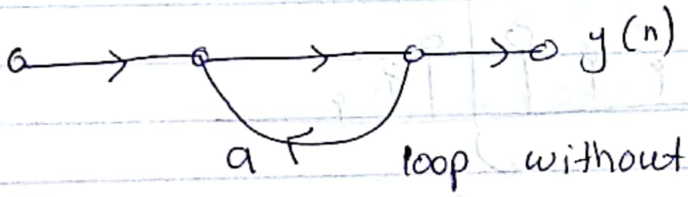




⇒ loop but h(n) is finite

* **Non Computable Network:**

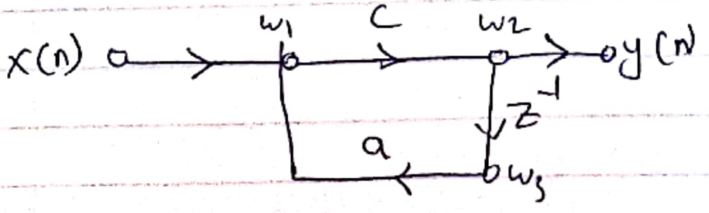
e.g: $y(n) = ay(n) + x(n)$



loop without z^{-1}

~~Transfare~~ **transposed form**

↳ Reverse direction of all Branches
↳ Inter change Input and output.

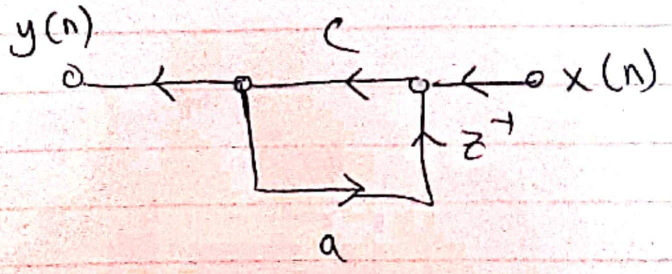


$Y(z) < (a z^{-1} y + X(z$

$$H(z) = \frac{c}{1-(az^{-1}}$$

✓

⇒



direct form II ما في ال ←

- دائما stable
- ممكن انه يكون linear phase.

Basic structures for FIR system:

$$y(n) = \sum_{k=0}^{M} b_k x(n-k)$$

IIR:
↳ عدد محدود أقل
"order أقل"

e.g: $y(n) = x(n) - x(n-3)$

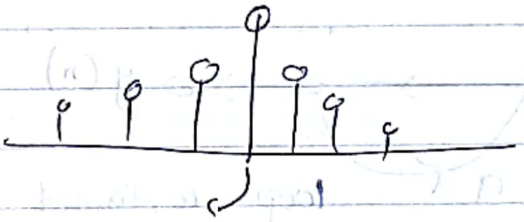
$h(n) = \{1, 0, 0, -1\}$.

- cascade forms:

- Implementation of linear phase FIR system 8=11

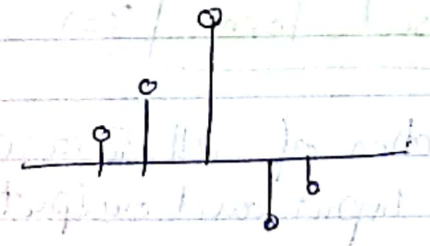
$h(n) = h(M-1-n)$

OR



$h(n) = -h(M-1-n)$ as mirror (symmetry condition)

↲ مختلفات في الاتجاه.



CH4: Sampling of continuous time Signal.

* **periodic Sampling.**

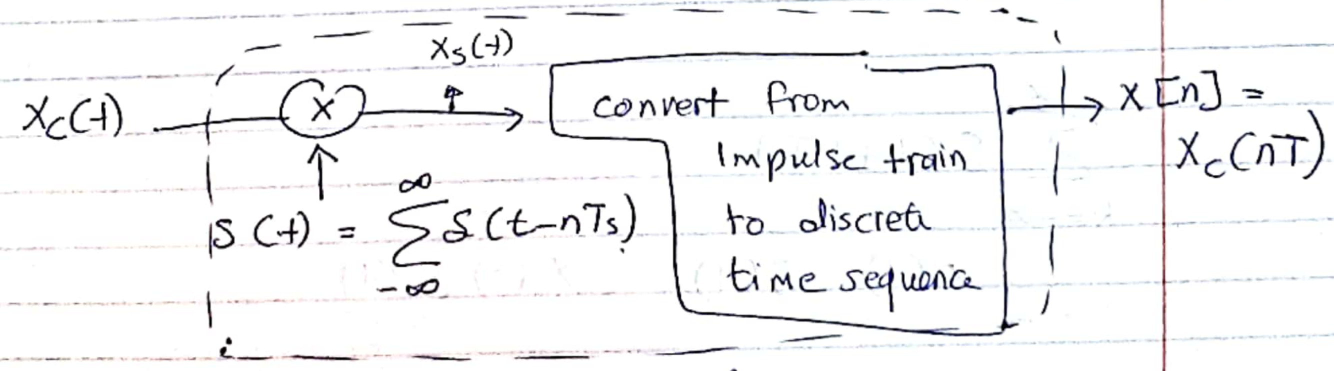
- quantization level (L) = $2^{\#\text{of bit}}$.

- Ts: sampling period.
- Fs: sampling rate Hz
- $\Omega_s = \frac{2\pi}{T}$ (Sampling frequency) rad/s

* best Fs ⇒ Nyquist rate is presented
↳ ≥ 2 (maximum freq. of signal)

⇒ $F_s \geq 2BW$

if $F_s <$ Nyquist rate ⇒ aliasing in Samplin



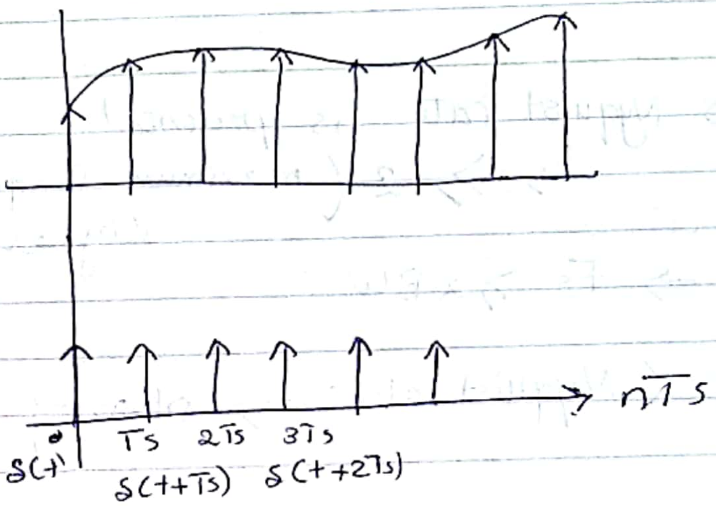
C/D converter.

Sampling process

↳ Two stages 3=))

① Impulse train modulator
(multiply the CT signal by train of Impulses)

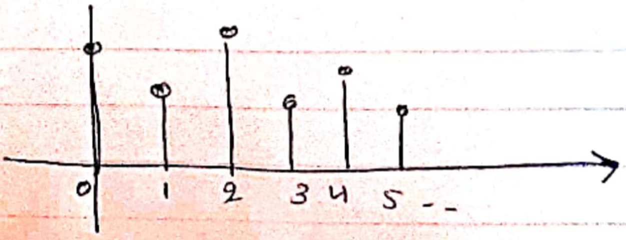
② Conversion of the Impulse train to sequence
[normalization divide by Ts]
In time domain



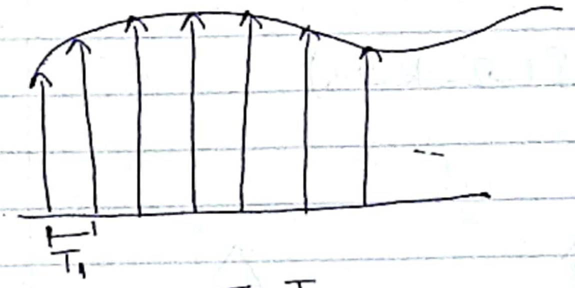
$$\sum_{k=-\infty}^{\infty} \delta(t+kT_s)$$

$$x(t) \cdot \delta(t) = x(0)\,\delta(t)$$

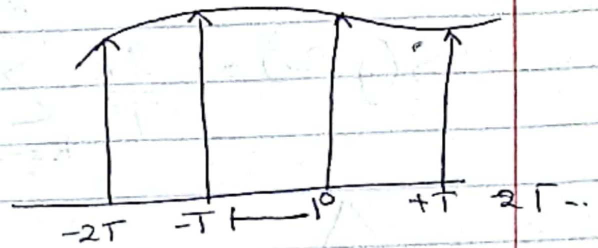
⇒ normalization $\frac{nT_s}{T_s}$



CT signal is the envelope of Impulse train



$T = T_1$



$T = 2T_1$

- freq. domain Representation of Sampling

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$x_s(t) = x_c(t) \, . \, s(t)$$
$$= x_c(t) \, . \sum_{n=-\infty}^{\infty} \delta(n-nT)$$
$$= \sum_{n=-\infty}^{\infty} x_c(nT) \; \delta(n-nT)$$

$$F\{x_s(t)\}$$

$$= F\{\underset{\substack{\downarrow \\ \text{multiplication to} \\ \text{Convolution}}}{x_c(t) \, . \, s(t)}\}$$

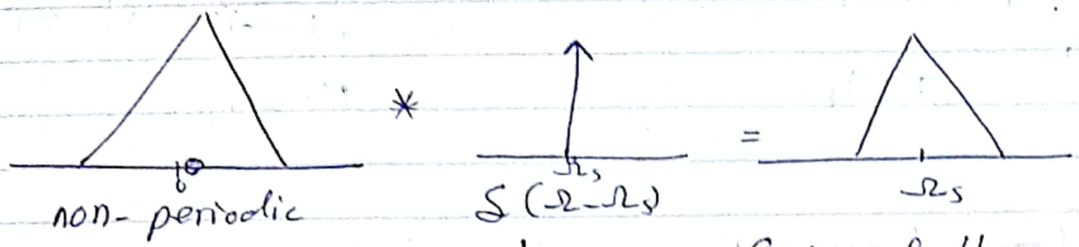
⇒ is a convolution of FT of $X_c(j\Omega)$ and $S(j\Omega)$

↳ $\Omega = 2\pi f$ rad/s

↳ $\omega = 2\pi \frac{f}{f_s}$ rad

Fourier transform of aperiodic Impulse train is aperiodic Impulse train

$$S(j\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s)$$



↳ many Copies of the signal spectrum

$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$$

↳ ~~Conv~~ Continues Convolution

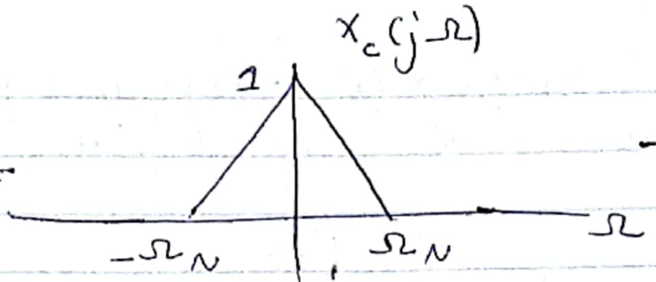
$$\boxed{\Omega_s = \frac{2\pi}{T}}$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s))$$

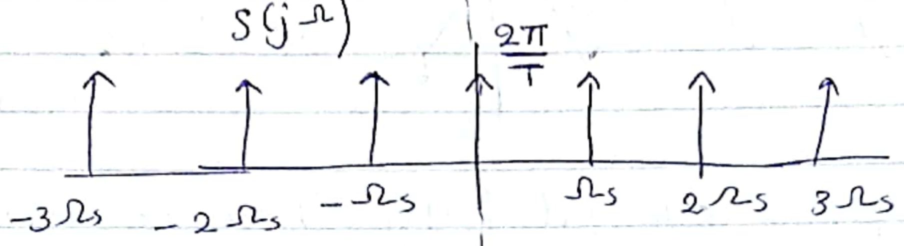
Periodically repeated copies of $X_c(j\omega)$

* Copies of $X_c(j\Omega)$ are shifted by integer Multiples of Sampling frequency Ω_s.

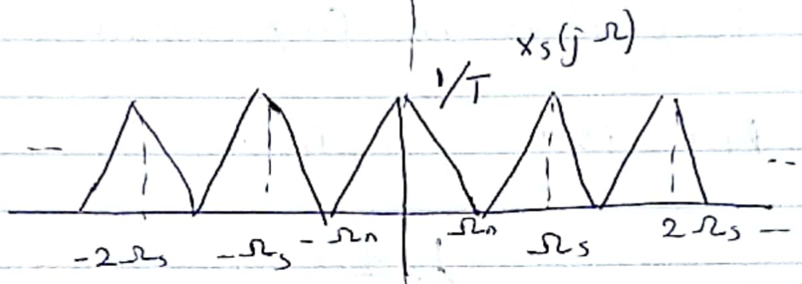
$2\,BW = 2\Omega_s \Leftarrow$



Ω_n: the highest frequency in $X_c(t)$



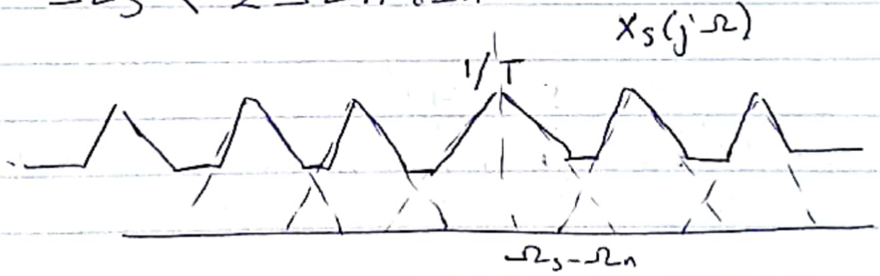
spectrum of Sampling function



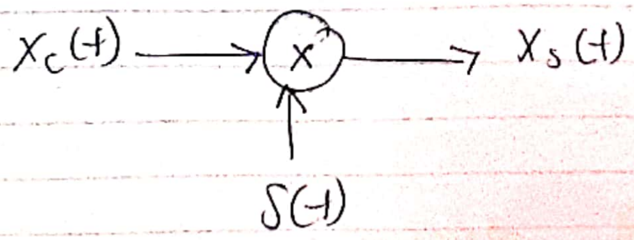
spectrum of Sampled signal

$\Omega_s > 2\Omega_n$

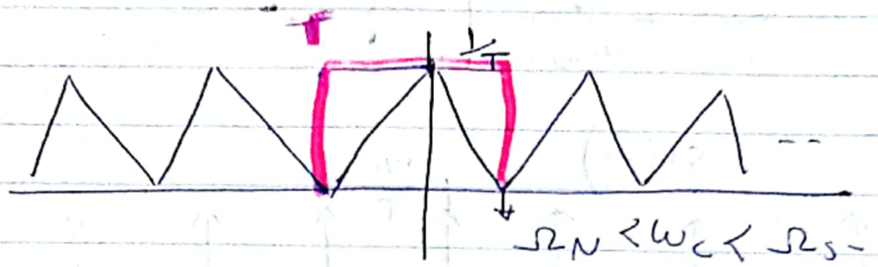
if $\Omega_s < 2\Omega_n$:=))



when $\Omega_s - \Omega_N > \Omega_n \Rightarrow \Omega_s > 2\Omega_s$
no overlaping



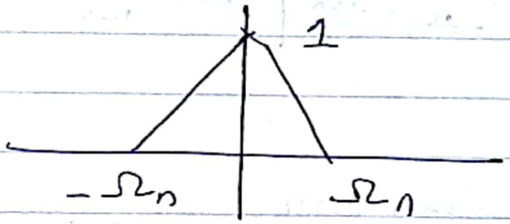
if we multiply $X_s(t)$ with analog LBF, then we can get the original $x_c(t)$.

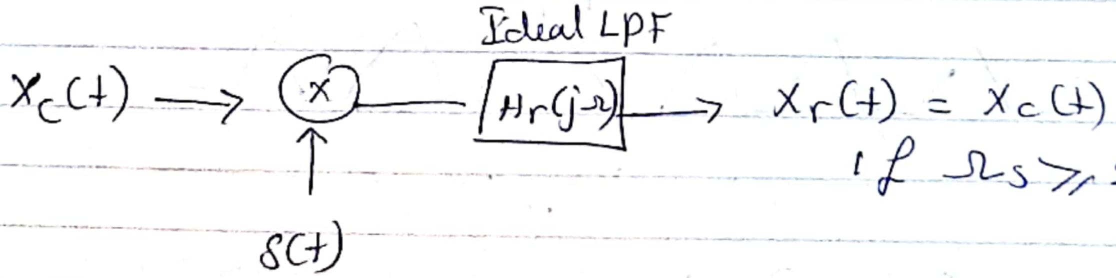


$\Omega_N < \omega_c < \Omega_s - \Omega_N$

analog not adigital filter because digital filtar is periodis in freq. domain

✱ analog LBF is non periodic.

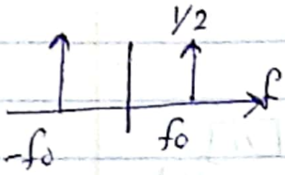


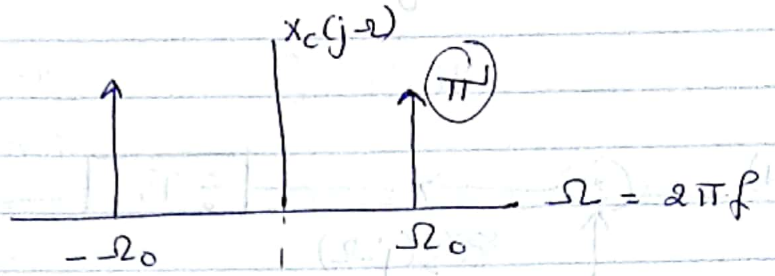


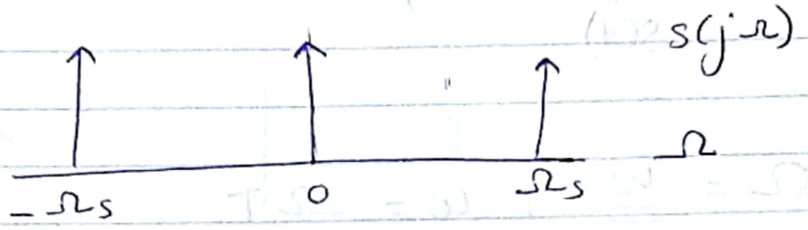
if $\Omega_s \geq 2\Omega_N$

periodic in time domain
not periodic in freq. domain

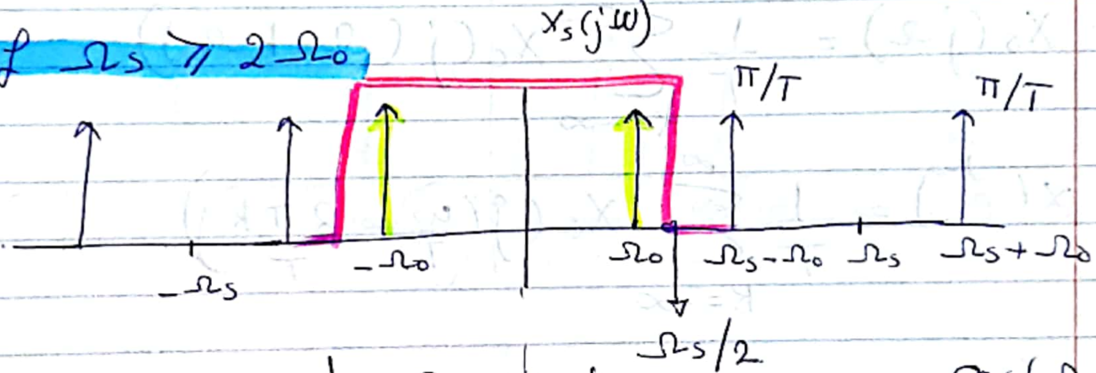
example 8 =)) Assume $X_c(t) = \cos(\Omega_0 t)$







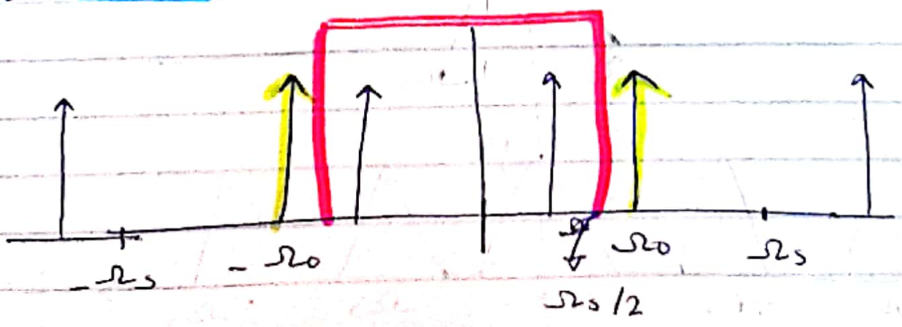
If $\Omega_s \geq 2\Omega_0$



↳ Reconstruction filter $\Omega_s = \Omega_s/2$ $\cos(\Omega_0 t)$

⇒ no aliasing.

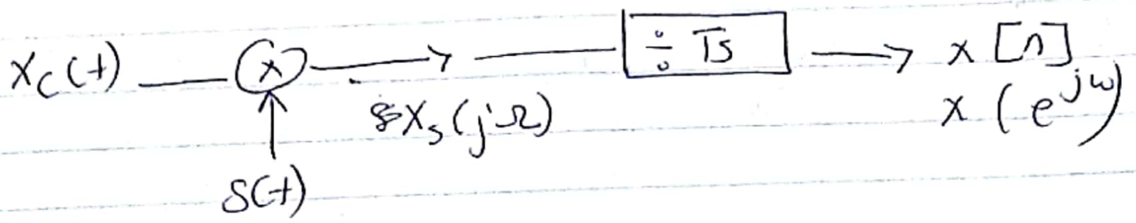
If $\Omega_s < 2\Omega_0$



$\cos((\Omega_s - \Omega_0)t)$

aliasing

The second stage is the normalization



$\Omega = \frac{w}{T}$, $w = \Omega T$

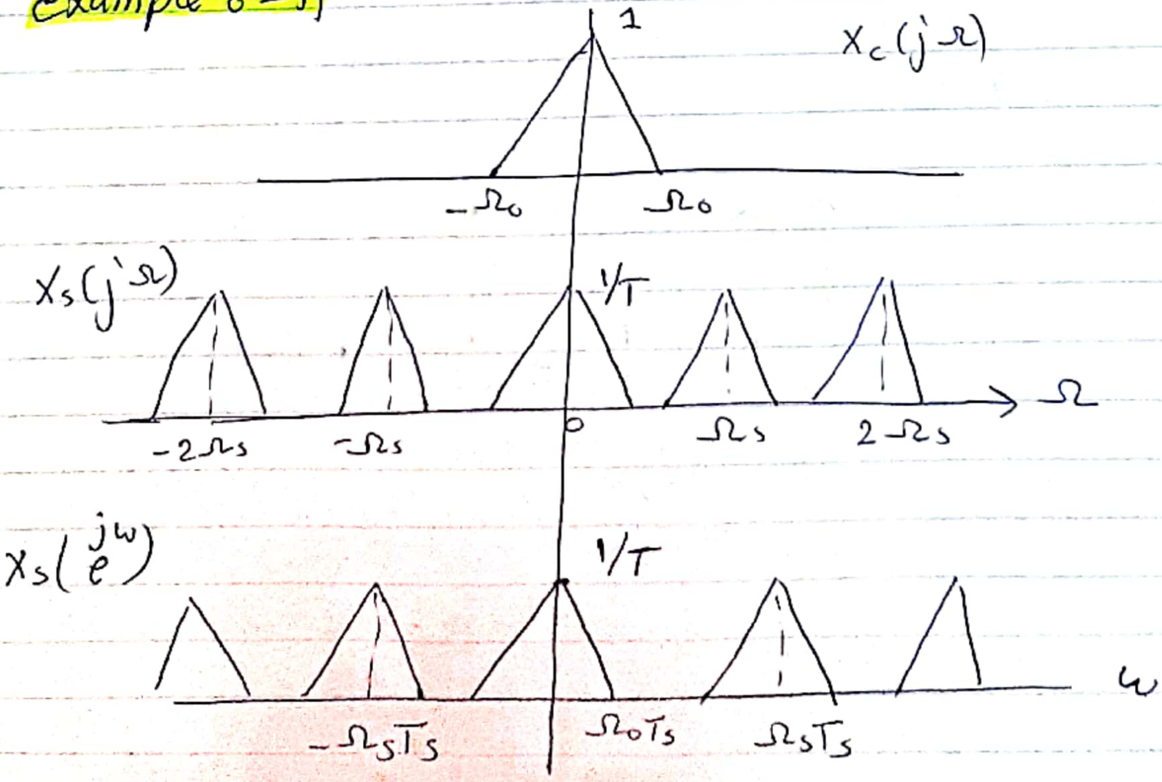
~~$x[n] =$~~

$x[n] = x_c(nT)$.

$$X_s(j\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s))$$

$$X(e^{jw}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{w}{T} - \frac{2\pi k}{T}\right)\right)$$

example :=))



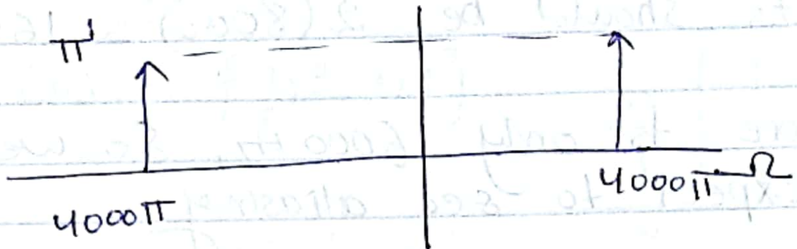
example: $x_c(t) = \cos(4000\pi t)$; $T = \frac{1}{6000}$ sec.

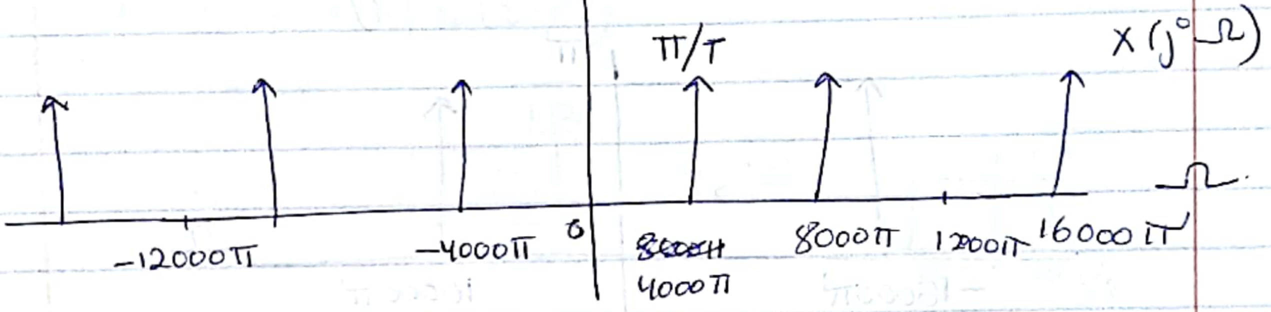
$f_0 = 2000\text{KHz}.$ $f_s = 6\text{KHz}.$

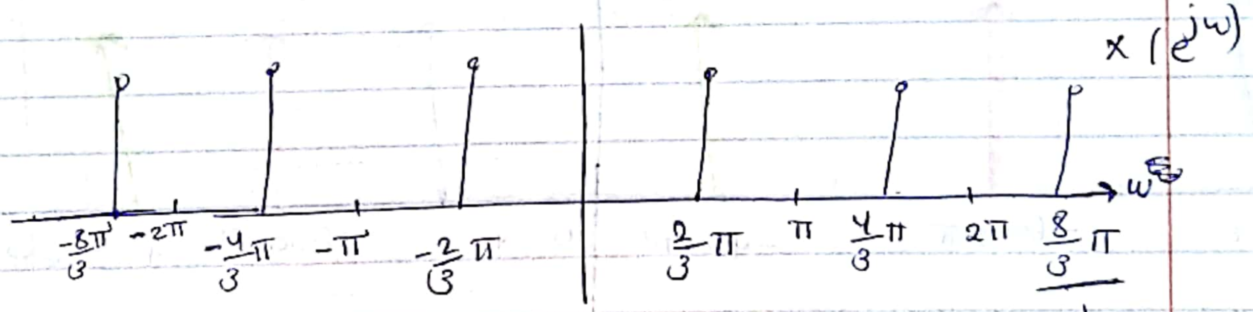
$\Omega = 4000\pi$

$\omega = T\Omega$ $\Omega_s = \frac{2\pi}{T} = 12000\pi$

$\omega_0 = \frac{2}{3}\pi$







$\downarrow$

$\frac{16000\pi}{6000\pi}$

$\boxed{\omega = T\Omega}$

example: $x_c(t) = \cos(16000\pi t)$, sampling period $T = \frac{1}{6000}$

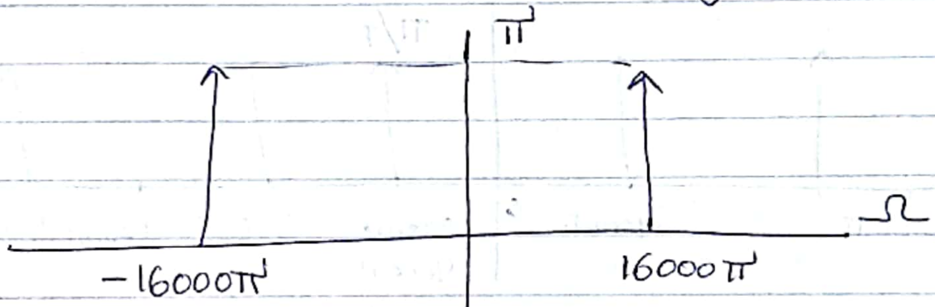
$\Omega = 16000\pi$

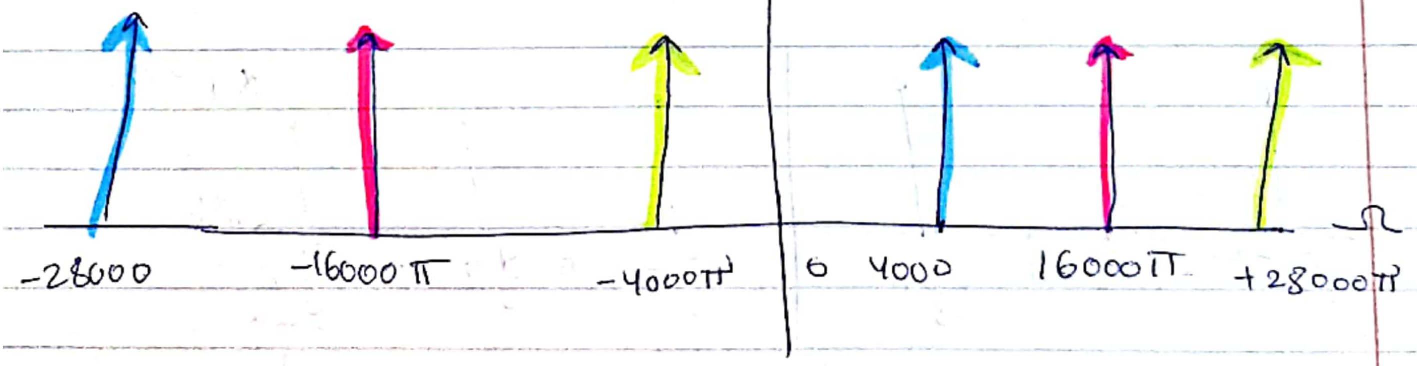
$f_0 = 8000$ Hz

$f_s = 6000$ Hz

⇒ So F_s should be $2(8000) = 16000$ Hz.

but here fs only 6000 Hz, So we expect to see aliasing.





$12000\pi + 16000\pi = 28000\pi$

$12000\pi - 16000\pi = -4000$

$-12000\pi + 16000\pi = 4000\pi$

$-12000\pi - 16000\pi = -28000\pi$

⇒ So, if we apply a filter, we will get $x_r(t) = \cos(4000\pi t)$, and it is not equal to $x_c(t)$.

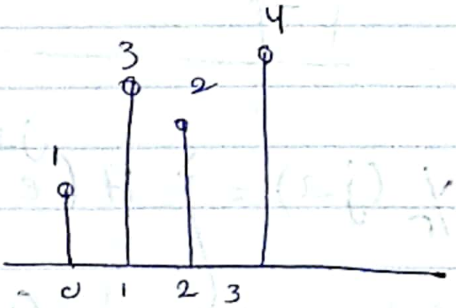
Reconstruction of BW signal form it's samples

D/C converter

$$x[n] \longrightarrow x_c(t)$$

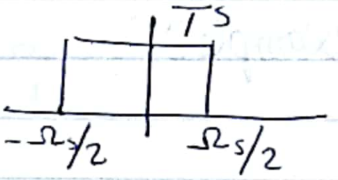
1) $$x_s(t) = \sum_{n=-\infty}^{\infty} x[n]\,\delta(t-nT)$$

$$x[n] = \{\underset{\uparrow}{1}, 3, 2, 4\}$$

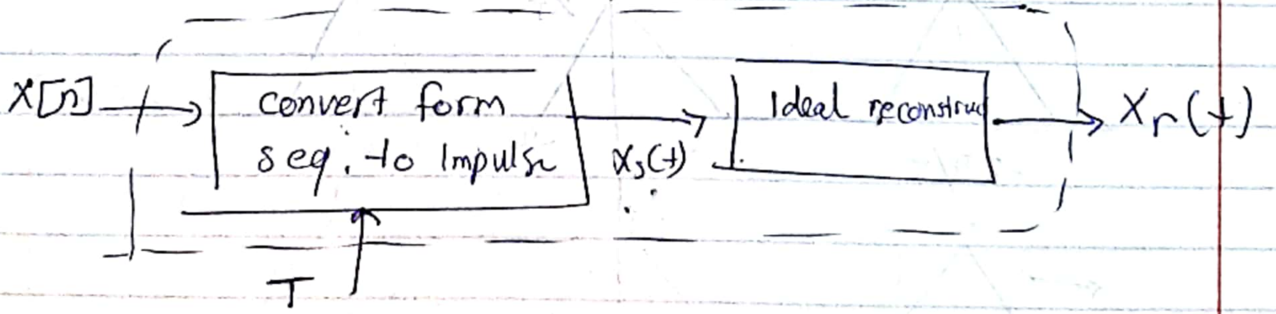


2) $$x_r(t) = \sum_{n=-\infty}^{\infty} x[n]\,h_r(t-nT)$$

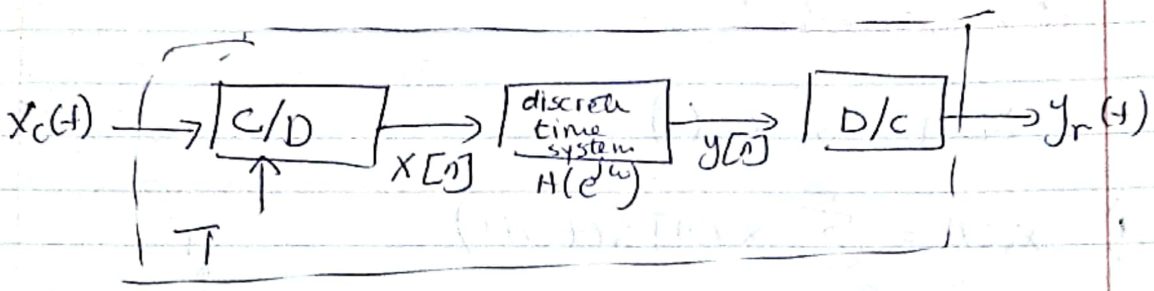
$$\begin{tikzcd} X_s(t) \arrow[r] & \boxed{h_r(t)} \arrow[r] \arrow[phantom, u, "\text{LBF}"] & X_r(t) \end{tikzcd}$$



* Ideal reconstruction system

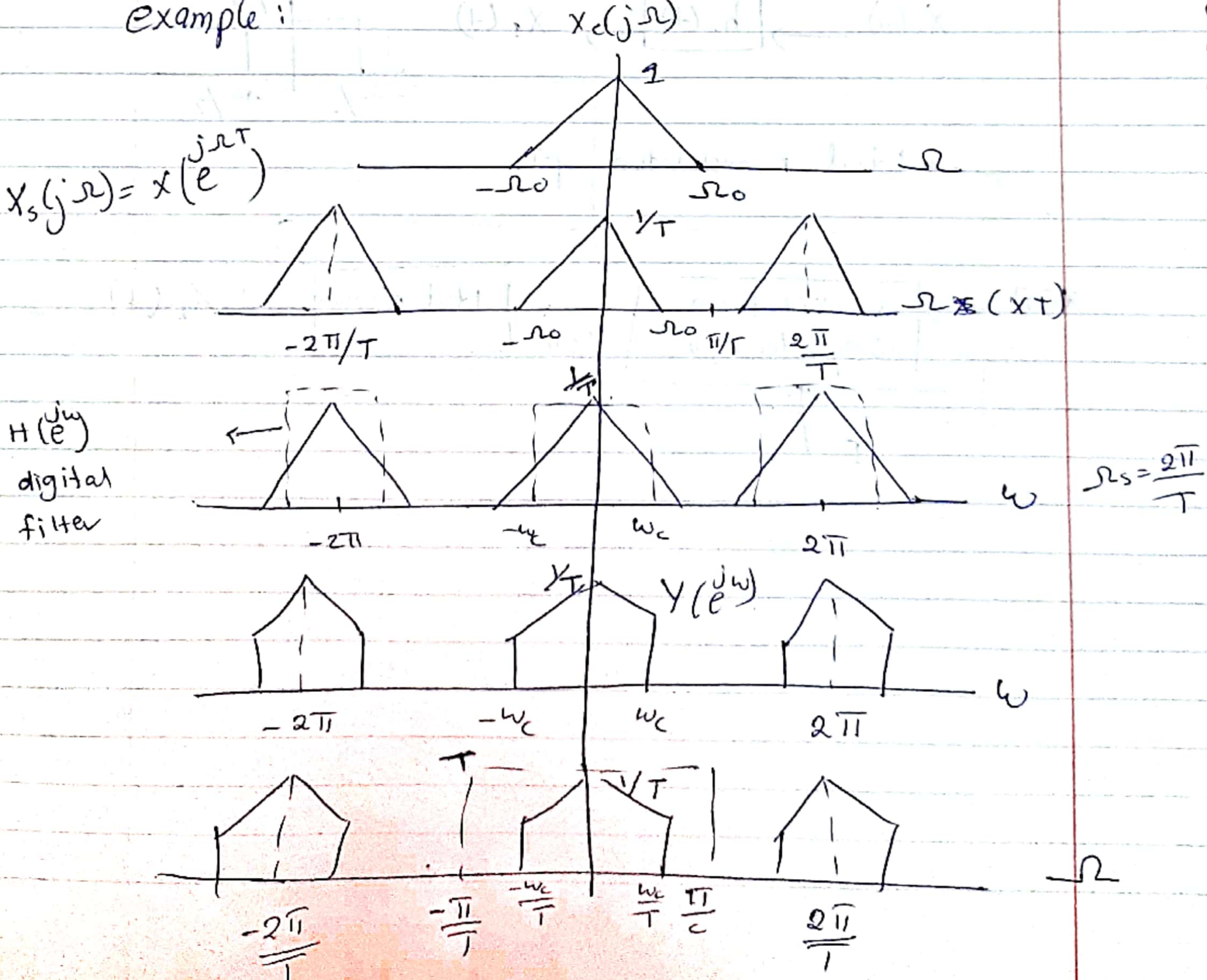


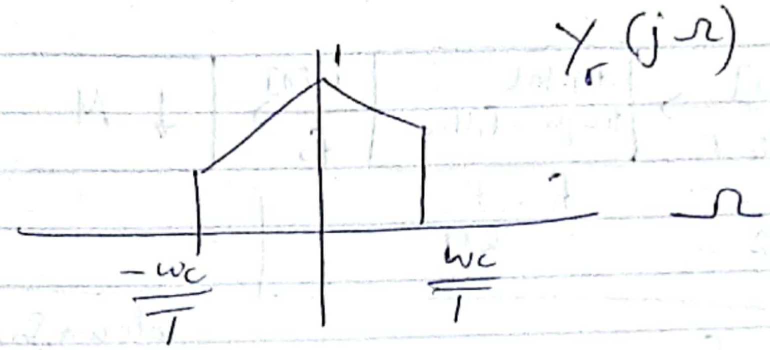
discrete time processing of Cont. time signal



$$Y_r(j\Omega) = \begin{cases} H(e^{j\Omega T})\, X_c(j\Omega) & , -\frac{\Omega_s}{2} < \Omega < \frac{\Omega_s}{2} \\ 0 & , o.w \end{cases}$$

example :





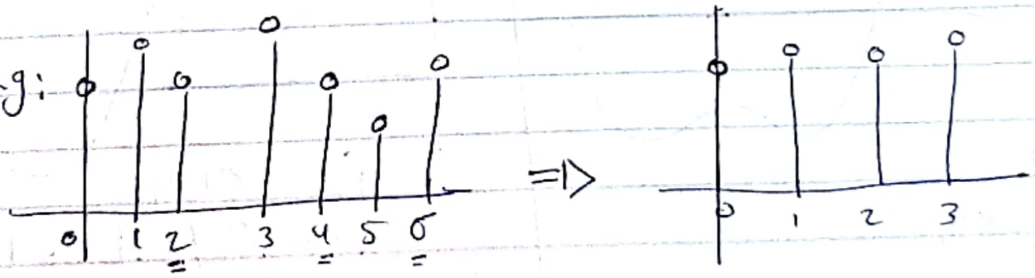
decimation 3=11

Is the two-step process of LPF followed by an operation knows as down Sampling

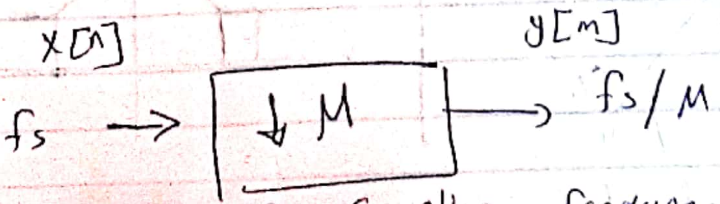
↳ we can down Sample the sequence of Sampled signal values by a factor of M

$$f_{s,new} = \frac{f_{s,old}}{M}$$

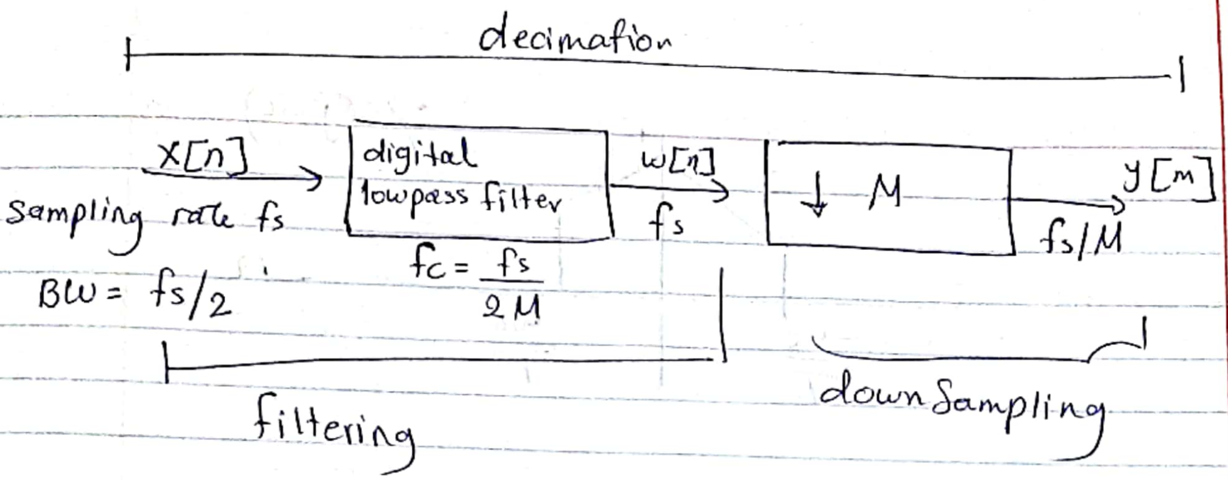
e.g:



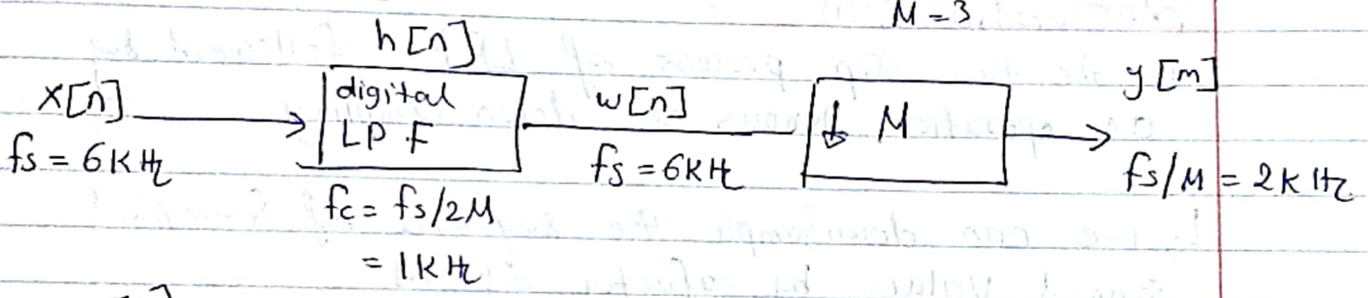
if $f_{s,old} = 8$ KHz and $\boxed{M = 2}$
then $f_{s,new} = 8/2 = 4$ KHz

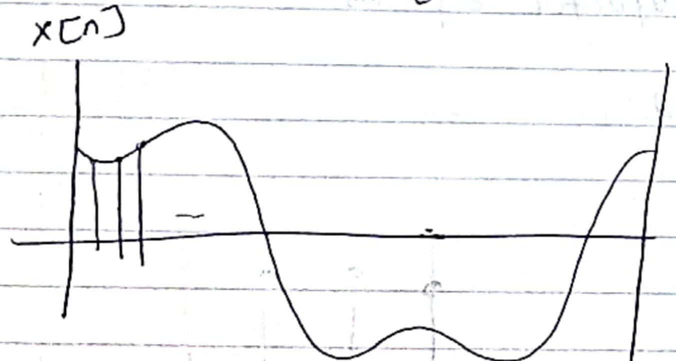


f_s: Sampling frequency

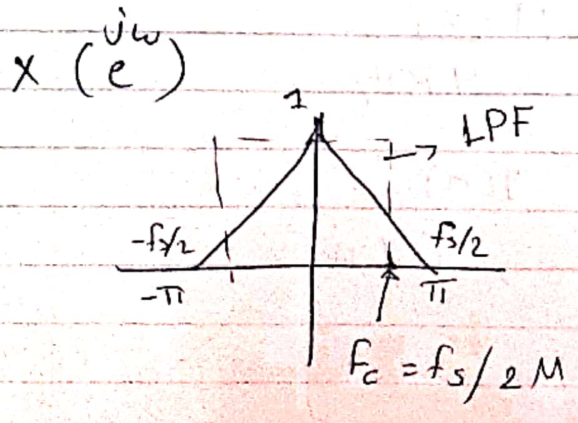


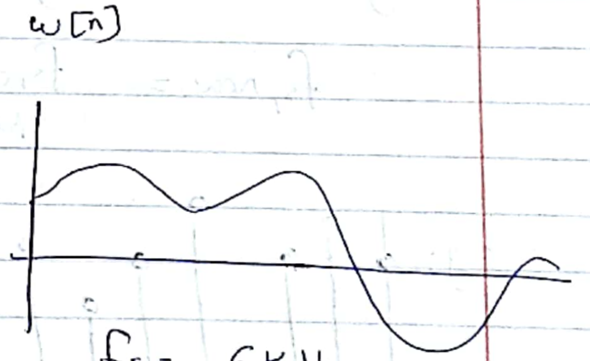
example (1)



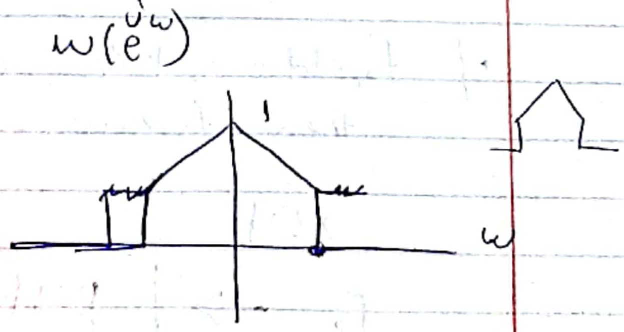


fs = 6KHz
B = 3KHz.

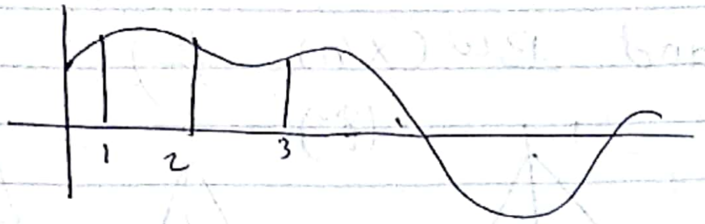




fs = 6KHz
BW = fs/2M = 1KHz



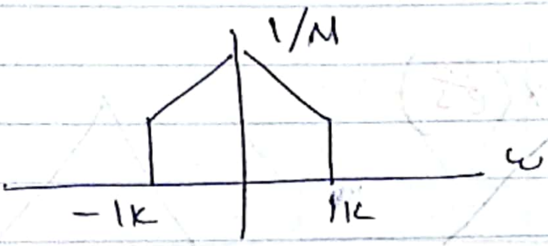
$Y[m]$



$f_s = \frac{6}{3} = 2\,kHz$

$B = 1\,kHz$

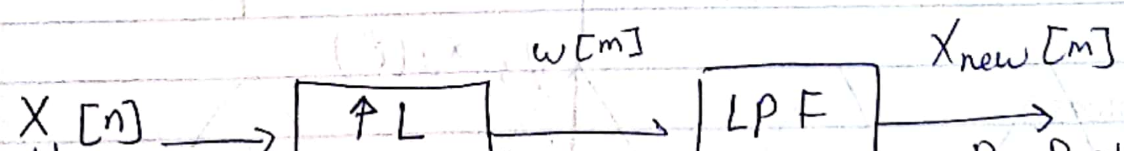
$Y(e^{jw})$:



زيادة عدد ال sample.

Interpolation :=))

Sample rate increses by Interpolation. New Sample values need to calculated.



$f_{s,new} = L\,F_{s,old}$

$B = B_{,old}$

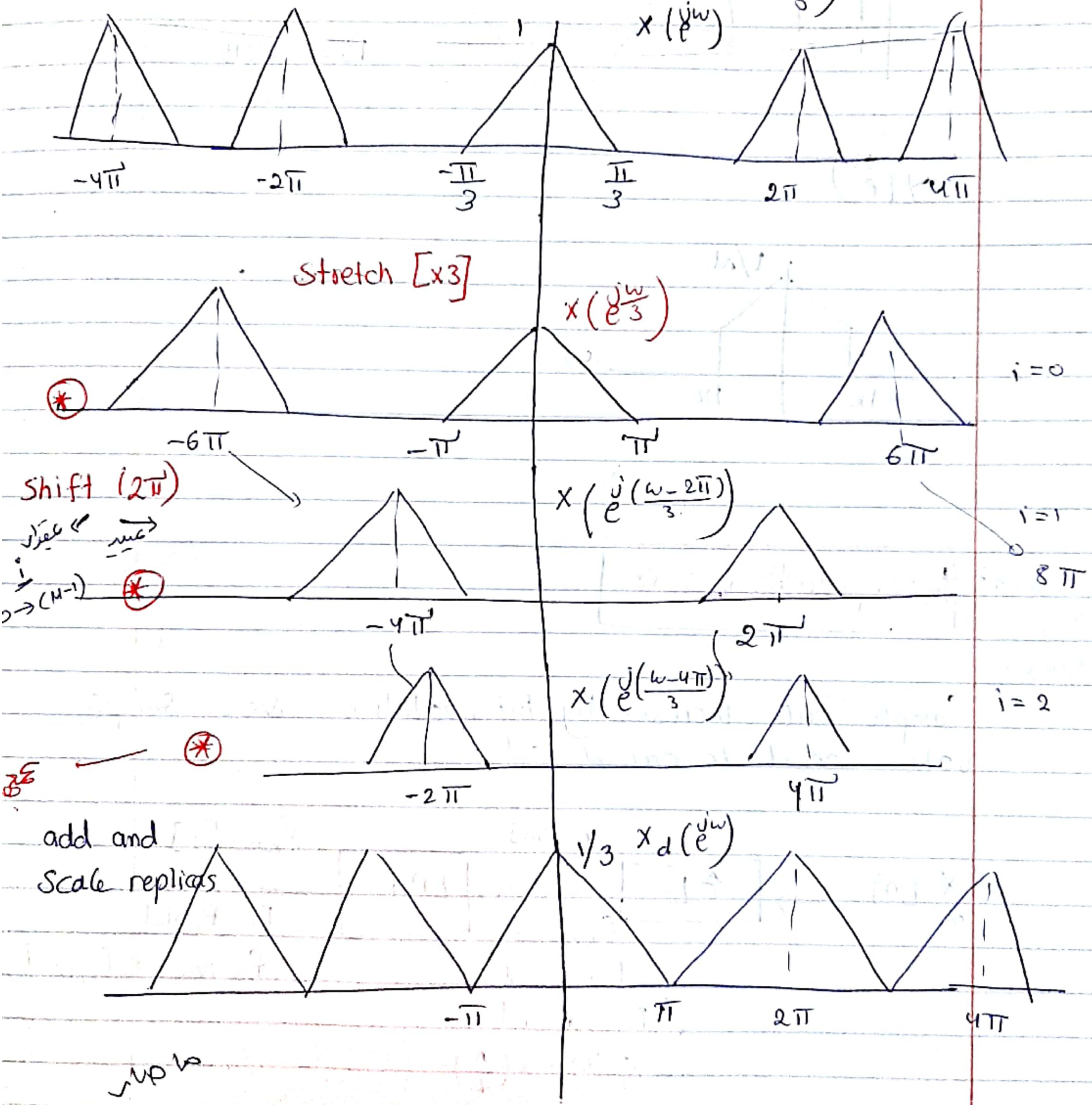
$\hookrightarrow X_L(n) = x\left(\frac{n}{L}\right)$

$B = B_{,old}$

$f_{s,new} = L\,f_{s,old}$

example (decimation)

let $M = 3$, and $BW(x(n)) = \frac{\pi}{3})$



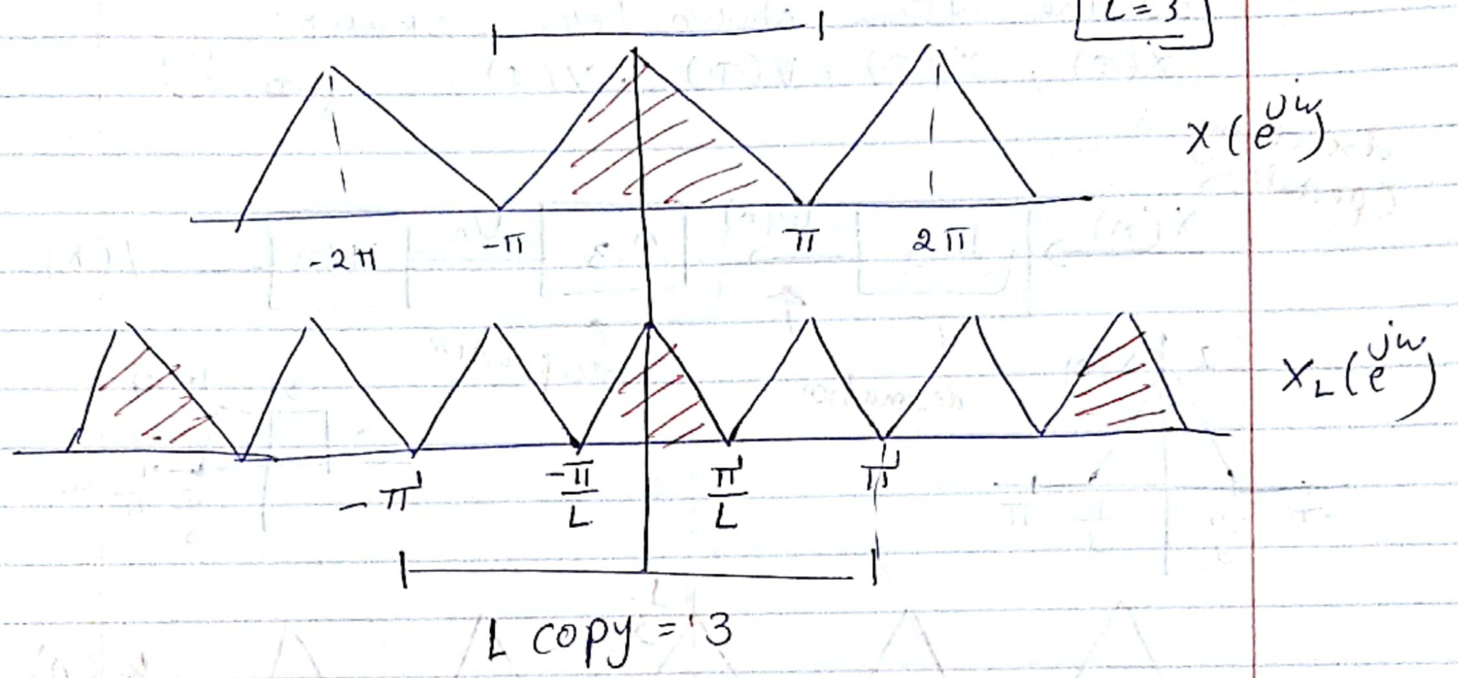
صار في
aliasing

$\to$ no aliasing

$\boxed{\text{Interpolation :=))}}$

$X_L(e^{j\omega}) = X(e^{j\omega L})$ one copy

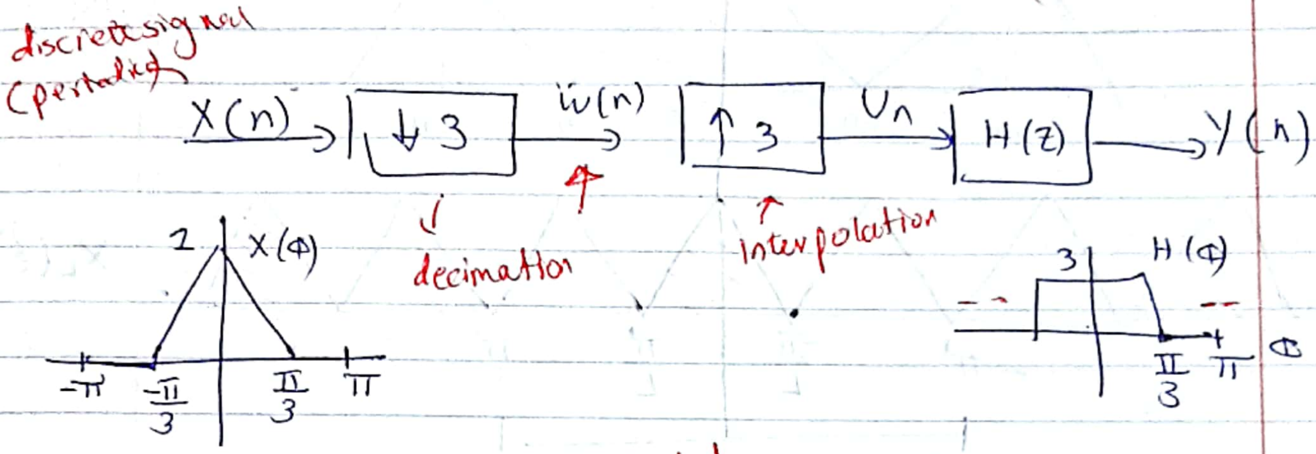
$\boxed{L=3}$

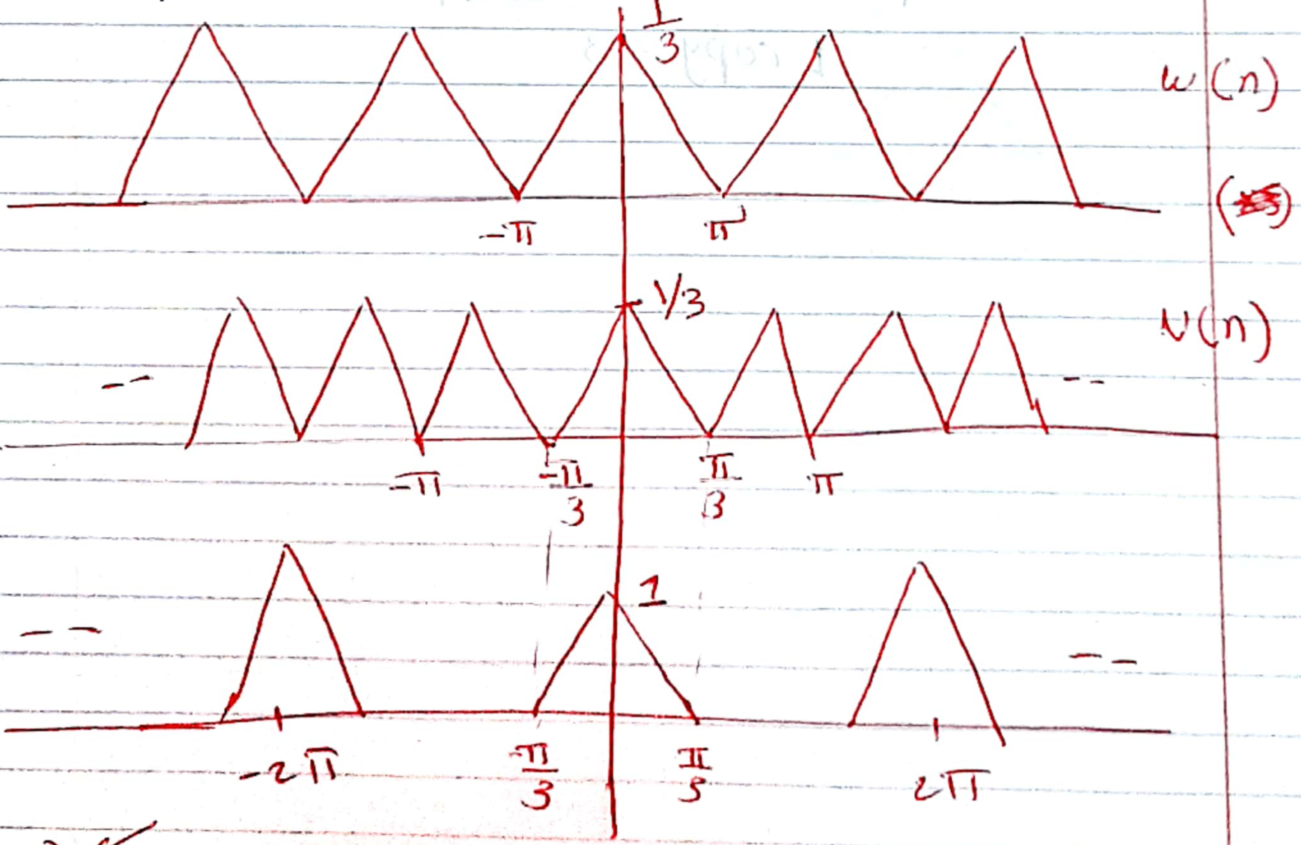


example:

an Input signal x(n) with spectrum x(Φ) is shown below, the Input signal is applide to the system shown below, sketck:

x(Φ) , x(Φ) , v(Φ) , y(Φ) $(\Phi : e^{jw})$

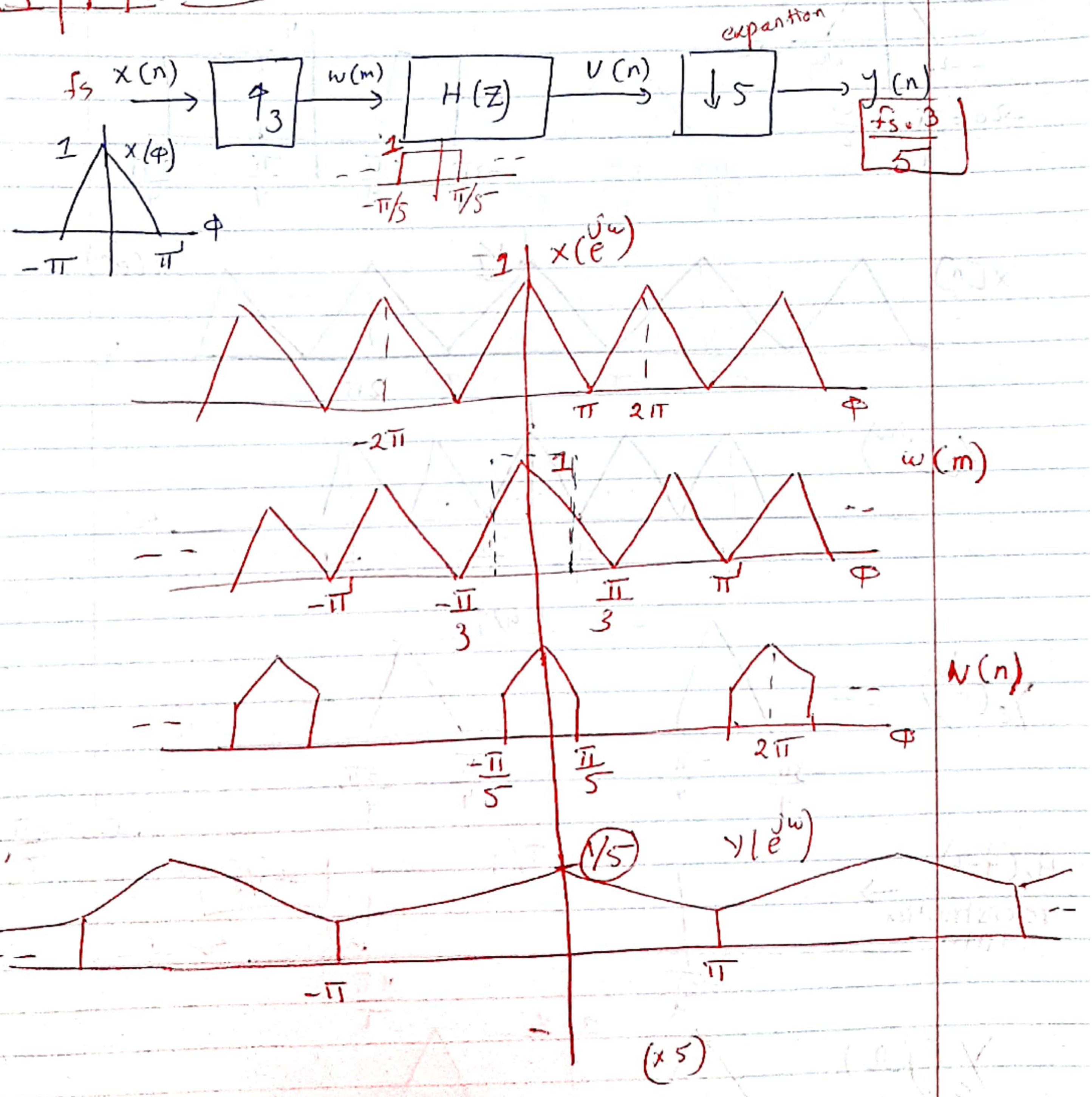




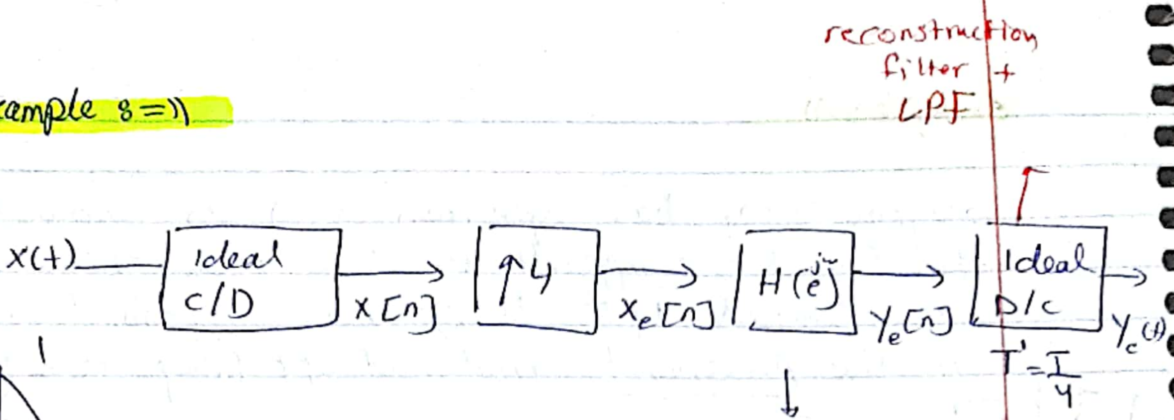
طلعت
يعني
$x(e^{jw})$

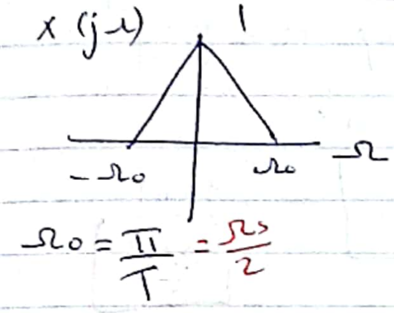
example 8=))

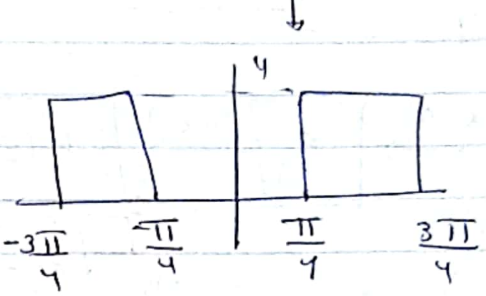
a signal x(n) has spectrum x(Φ) as show below, the Signal is applied to the system shown below, the ideal LPF H(z) has again factor of 1 in the passband and cut-off freq. $\Phi = \frac{\pi}{5}$

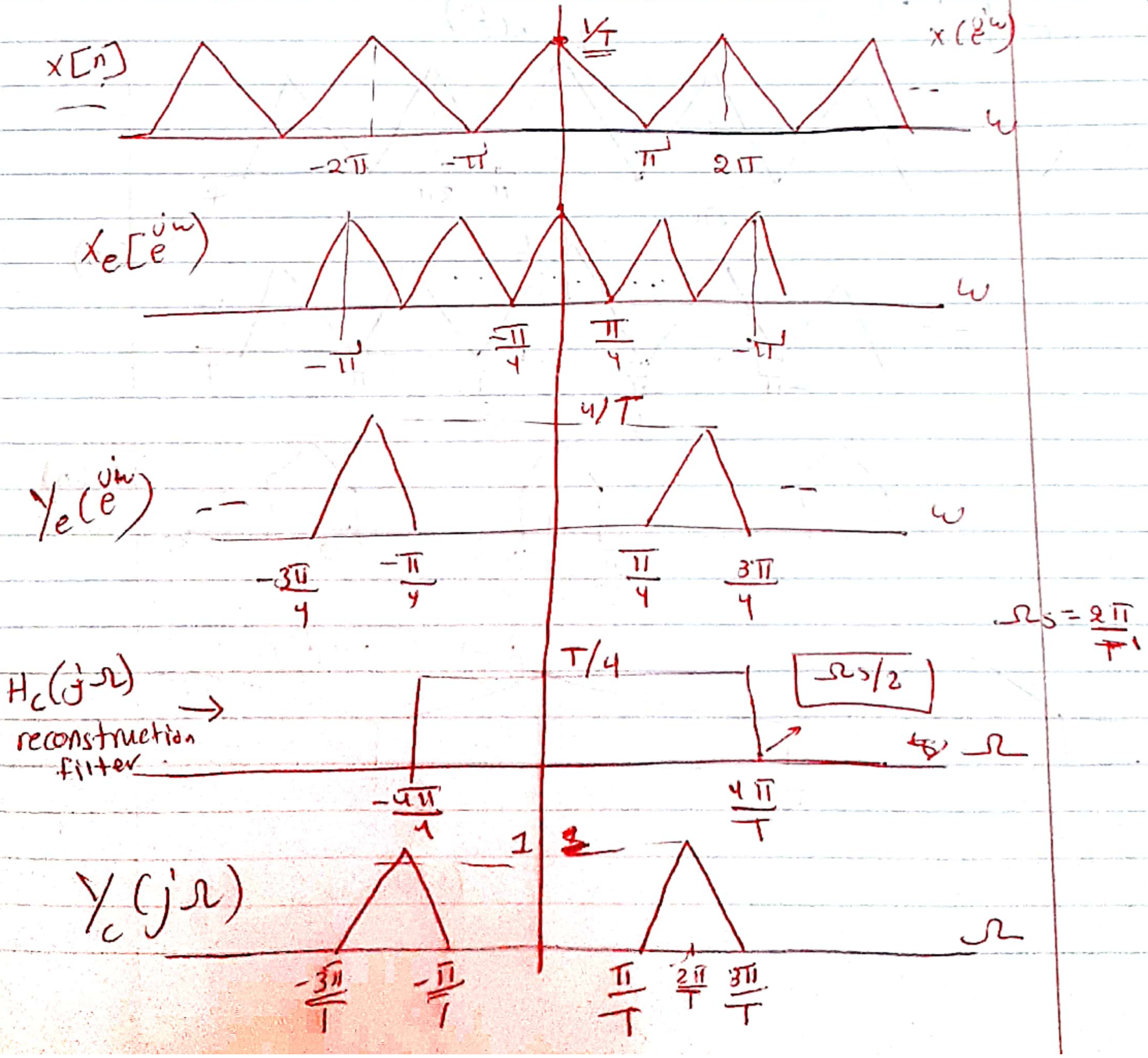


example 8 =))

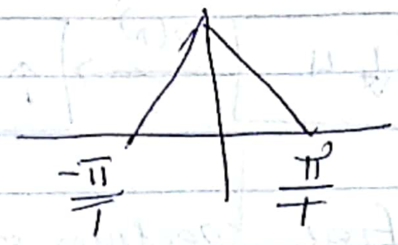

reconstruction filter + LPF
x(t)
Ideal C/D
x[n]
↑4
$x_e[n]$
$H(e^{jw})$
$y_e[n]$
Ideal D/C
$y_c(t)$
$T' = \frac{T}{4}$


$X(j\Omega)$
1
$-\Omega_0$
Ω_0
Ω
$\Omega_0 = \frac{\pi}{T} = \frac{\Omega_s}{2}$


4
$-\frac{3\pi}{4}$
$-\frac{\pi}{4}$
$\frac{\pi}{4}$
$\frac{3\pi}{4}$


$X[n]$
$X(e^{jw})$
1/T
w
-2π
$-\pi$
π
2π
$X_e[e^{jw})$
w
$-\pi$
$-\frac{\pi}{4}$
$\frac{\pi}{4}$
π
4/T
$Y_e(e^{jw})$
w
$-\frac{3\pi}{4}$
$-\frac{\pi}{4}$
$\frac{\pi}{4}$
$\frac{3\pi}{4}$
$\Omega_s = \frac{2\pi}{T'}$
T/4
$H_c(j\Omega)$
reconstruction filter
$\Omega_s/2$
Ω
$-\frac{4\pi}{T}$
$\frac{4\pi}{T}$
1
$Y_c(j\Omega)$
Ω
$-\frac{3\pi}{T}$
$-\frac{\pi}{T}$
$\frac{\pi}{T}$
$\frac{2\pi}{T}$
$\frac{3\pi}{T}$

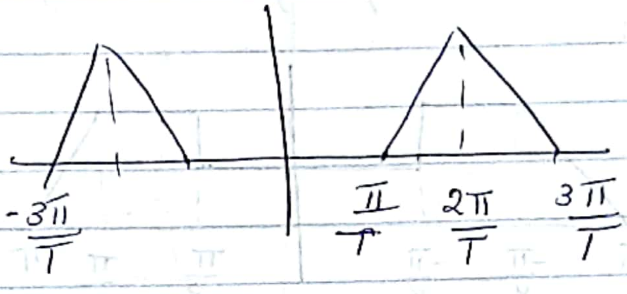
* العلاقة بين ال Input and ال output



$\Rightarrow$ shift by $\pm\frac{2\pi}{T}$

(In time domain) $\Rightarrow$

* cos

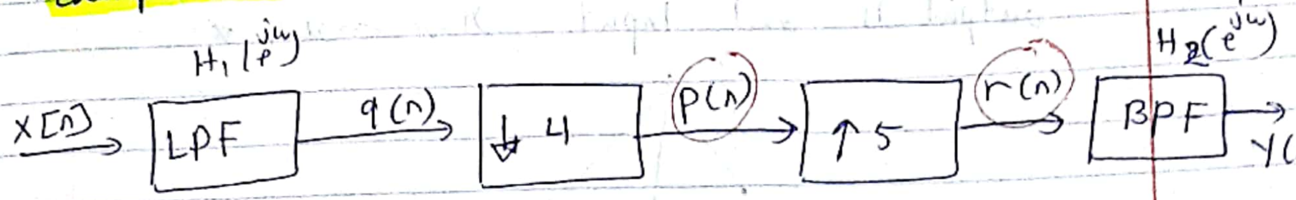


$$\Rightarrow Y_c(j\Omega) = X_c\left(j\left(\Omega - \frac{2\pi}{T}\right)\right) + X_c\left(j\left(\Omega + \frac{2\pi}{T}\right)\right)$$

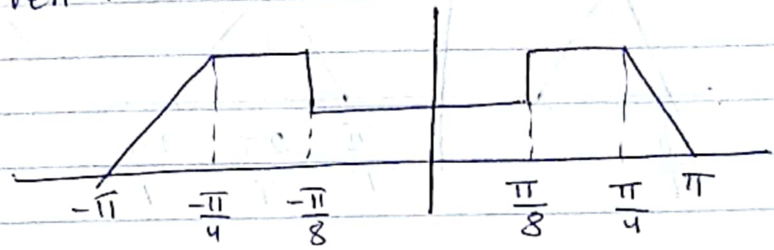
there for

$$y_c(t) = 2\,X_c(t)\cos\left(\frac{2\pi}{T}t\right)$$

example :-



If the magnitude freq. spectrum of x(n), $X(e^{jw})$ is given :



$$H_1(e^{jw}) = \begin{cases} 1, & |w| \leq \pi/4 \\ 0, & \text{o.w} \end{cases}$$

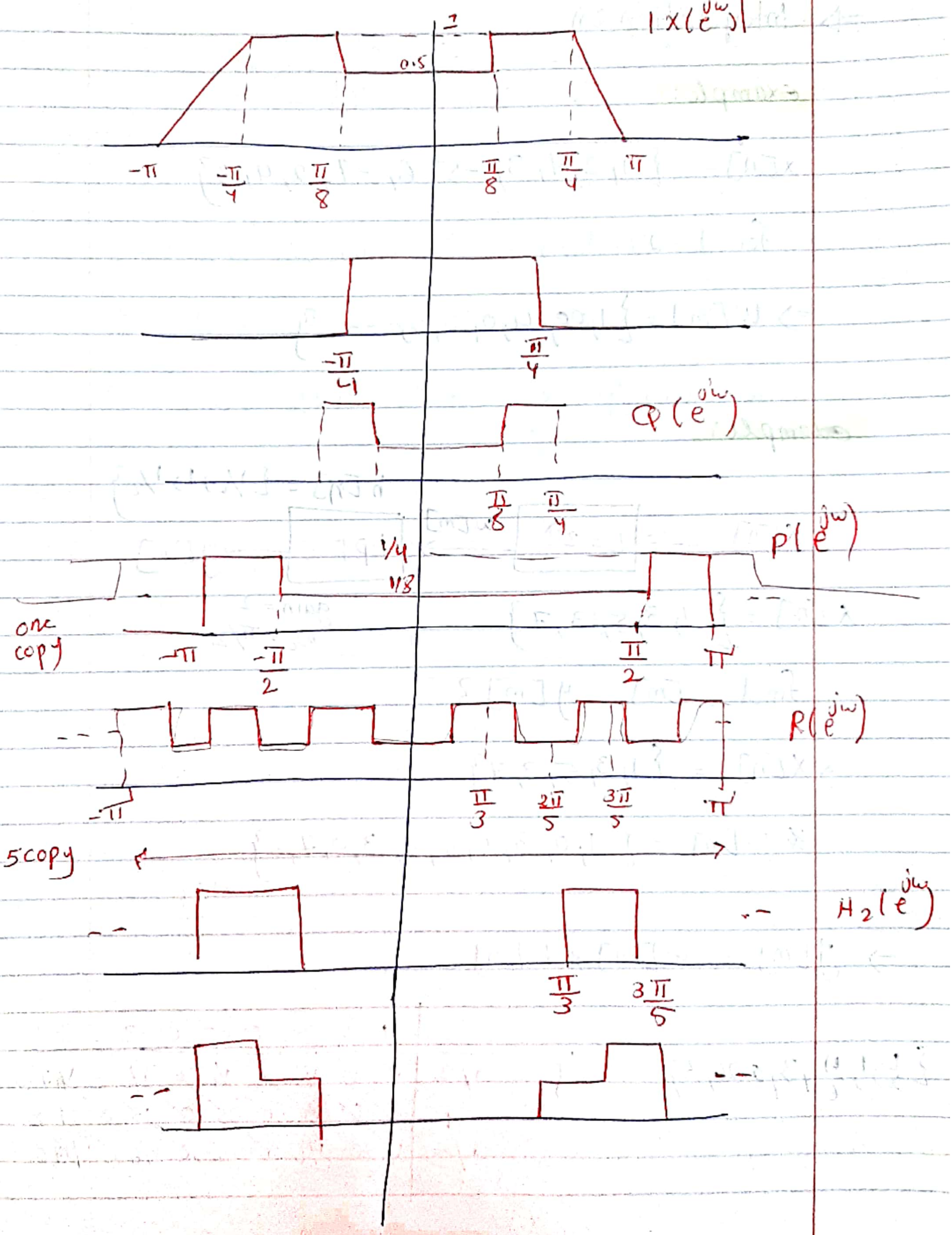
$$H_2(e^{jw}) = \begin{cases} 1, & \frac{\pi}{3} \leq |w| \leq \frac{3\pi}{5} \\ 0, & \text{o.w} \end{cases}$$

Sketch $Q(e^{jw})$

$P(e^{jw})$

$R(e^{jw})$

$Y(e^{jw})$



|X(e^{jw})|
1
0.5
-π
-π/4
π/8
π/8
π/4
π
-π/4
π/4
Q(e^{jw})
π/8
π/4
P(e^{jw})
1/4
1/8
one copy
-π
-π/2
π/2
π
R(e^{jw})
-π
π/3
2π/5
3π/5
π
5 copy
H_2(e^{jw})
π/3
3π/6

⇒ Interpolation :=))

example:

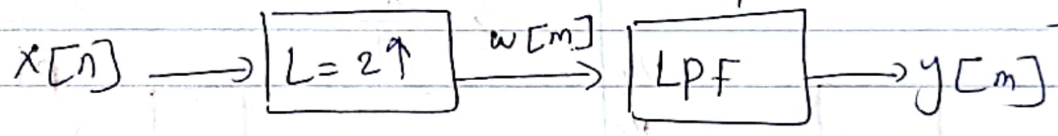
$x[n] = \{1, 2, 4, 3, -5, 6, -7, 2, 4, 3\}$

for $L = 2$:

$\Rightarrow w[m] = \{1, 0 0, 4, 0, 3, 0, -- \}$

example:

$h[n] = \{1/2, 1, 1/2\}$



gain = 2

$w_c = \pi/2$

$x[n] = \{1, 3, 5, 3, 7\}$

find $w[m]$, $y[m]$?

$\rightarrow x[n] = \{1, 3, 5, 3, 7\}$

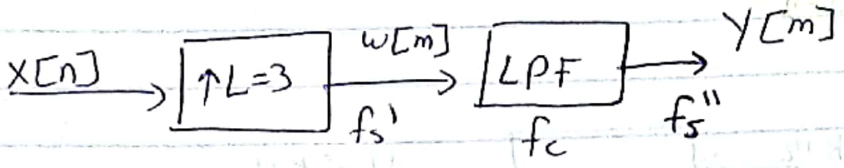
$* \; w[m] = \{1, 0, 3, 0, 5, 0, 3, 0, 7, 0\}$

$\rightarrow y[m] = w[m] * h[n]$

$\{\frac{1}{2}, 1, \frac{4}{6}, 3, 0.26, 5, -- \}$

	1	0	3	0	5	0	3	0	7	0
1/2	1/2	0	1/6	0	1/10	0	1/6	0	1/14	0
1	1	0	3	0	5	0	3	0	7	0
1/2	1/2	0	1/6	0	1/10	0	1/6	0	1/14	0

example:



$f_s = 8kHz$

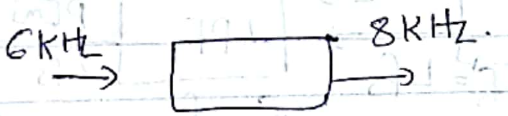
$BW = 2kHz.$

what are the value of fc and fs' ?

$f_s'' = f_s' = Lf_s = 3(8k) = 24kHz.$

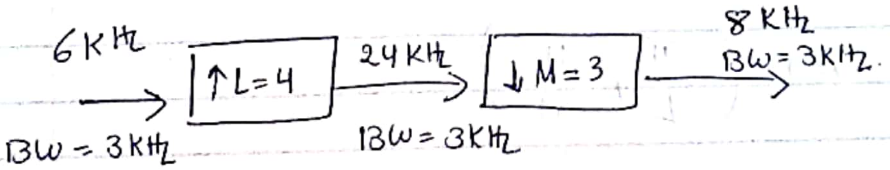
$f_c = BW = 2kHz.$ OR $\frac{f_s}{2}$ if BW is not given.

Sampling Rate conversion by non Integer factor 3=1)



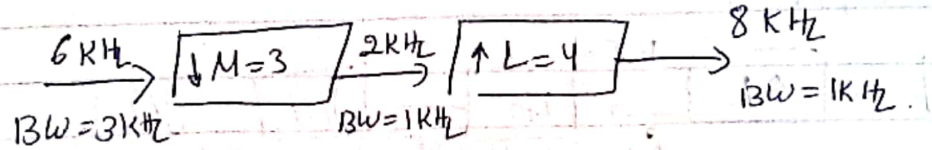
In this case L is non-integer so we need to do the following steps:

1)



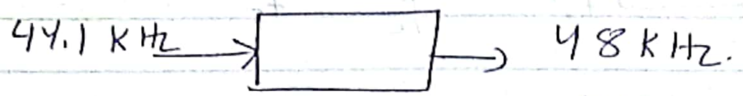
better ⇒ not change the BW

2)



the overall process is interpolation since the ratio is $\frac{4}{3}$ more than 1.

example :=))

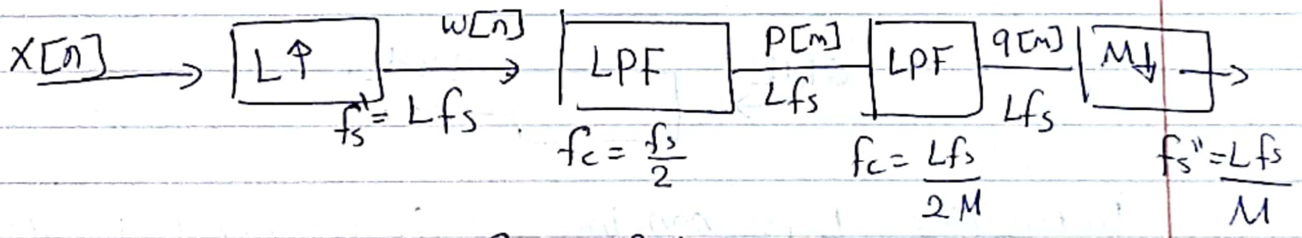


DAT
digital Audio Tape

CD
Compact dist

$$\frac{4800}{44110} = \frac{160}{147} \begin{matrix} \leftarrow L \\ \leftarrow M \end{matrix}$$

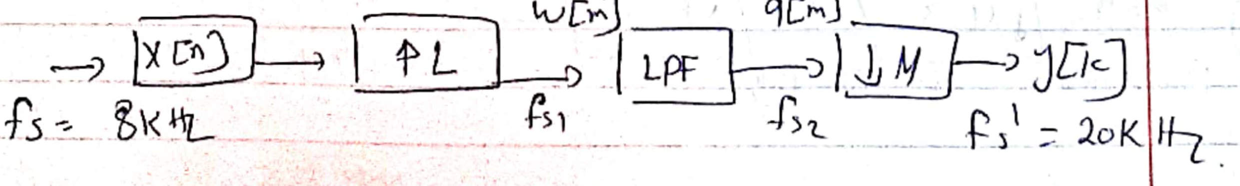
over all process :=))



$$f_c = \min\left(\frac{f_s}{2}, \frac{Lf_s}{2M}\right)$$

$$w_c = \min\left(\frac{\pi}{L}, \frac{\pi}{M}\right)$$

example: L, M = ?



$\frac{20}{8} = \frac{5}{2} = \frac{L}{M}$ / $f_{s_1} = 8(5) = 40$

$L > M \Rightarrow$ Interpolation $\Rightarrow$ BW not change = 4KHz.

decimation in freq. domain

$$X_s(j\Omega) = \frac{1}{T}\sum X_c(j(\Omega - n\Omega_s))$$

$$X_s(e^{j\omega}) = \frac{1}{T}\sum X_c\left(\frac{\omega}{T} - \frac{2\pi n}{T}\right)$$

$$x_d(n) = x(nM) = x_c(nT') \;;\; T' = MT$$

$$X_d(e^{j\omega}) = \frac{1}{MT}\sum X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi r}{MT}\right)\right)$$

$$= \frac{1}{M}\sum X_s\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi i}{M}\right)}\right)$$